Minimizing the Age of Synchronization in Power-Constrained Wireless Networks with Unreliable Time-Varying Channels

Qining Zhang[1], Haoyue Tang[1], Jintao Wang[1,2]
[1]Beijing National Research Center for Information Science and Technology (BNRist),
Dept. of Electronic Engineering, Tsinghua University, Beijing, China
[2]Research Institute of Tsinghua University in Shenzhen, Shenzhen, China
{zqn17@mails.;thy17@mails.;wangjintao@}tsinghua.edu.cn,

***Abstract*—We study a network with a central controller collecting random updates from power-limited sensors. The time-varying channels between sensors and the central controller are modeled as ergodic Markov chains while packet-loss may happen due to decoding error. We measure the data freshness from the central controller by the metric *Age of Synchronization* (AoS), i.e., the average time elapsed since information about a sensor becomes desynchronized. To minimize the average AoS under all aforementioned bandwidth and power constraints, we first relax the hard bandwidth limit and decouple the multi-sensor problem into a single-sensor constrained Markov decision process (CMDP), which is then solved through linear programming (LP). We then propose an asymptotic optimal scheduling policy to solve the original hard-bandwidth-constrained problem. It is revealed that sensors are more likely to send updates under better channel states and higher AoS to save energy and avoid packet-loss.**

I. Introduction

Nowadays, the unprecedented development of Internet technology has proliferated plenty of real-time applications including Internet of Things (IoT), Virtual Reality (VR), Augmented Reality (AR), etc. In these scenarios, update packets are generated randomly by the external environment due to alternation and mobility, and a central controller collects such random updates from a large number of sensors under wireless constraints like bandwidth and power consumption. The data synchronicity of the central controller directly determines the quality of service in such networks.

To measure data synchronicity from the perspective of the receiver, the metric *Age of Synchronization (AoS)* is proposed [1]. By definition, it shows the time elapsed since the information at the receiver is no longer synchronized with the source. Due to the difference between AoS and metric *Age of Information* (AoI) [2], AoI minimization strategies [3]–[7] cannot guarantee a good AoS performance. The analysis and design of AoS minimization strategies have been studied in [8]–[11]. However, the above analyses only assume time-invariant channels during transmission, which is impractical in wireless scenarios. Although the data desynchronization status in an error-prone network has been studied in [12], the goal in that work aims at minimizing the average number of desynchronized devices. The average time of desynchronization duration across the network has not been taken into account.

To analyze AoS performance and design scheduling strategies in a time-varying and error-prone wireless network, in our work, we consider a central decision-making unit (central controller) collecting information from multiple information-collecting units (sensors), as depicted in Fig. 1, where the communication channels connecting each sensor and the BS are quantized into discrete states and vary across slots. The goal is to design scheduling strategy that satisfies the power constraint of each sensor and the limited bandwidth shared by all the sensors. The contributions of the paper are as follows: we first formulate the optimization problem of minimizing average AoS under both bandwidth and power constraints in a multi-sensor time-varying network. Next, we decouple the multi-sensor scheduling problem into a single-sensor transmission problem by relaxing the hard bandwidth constraint. The single-sensor transmission problem can be formulated into a constrained Markov decision process (CMDP) and solved through linear programming (LP). Finally, we propose an asymptotic policy to solve the original scheduling problem with a hard bandwidth limit.

The rest of the paper is organized as follows: We introduce the system model, the metric AoS, and problem formulation in Section II. Then, we decouple the problem into a single-sensor CMDP and solve it through LP in Section III. In Section IV, we propose a multi-sensor scheduling algorithm and analyze its theoretic performance. The simulation results are provided in Section V and Section VI draws the conclusion.

II. System Model and Problem Formulation

A. System Model

In this work, we consider a network where a central controller collects information from N sensors tracking independent random external updates, as shown in Fig. 1. Consider a discrete-time scenario and use $t \in \{1, \cdots, T\}$ to

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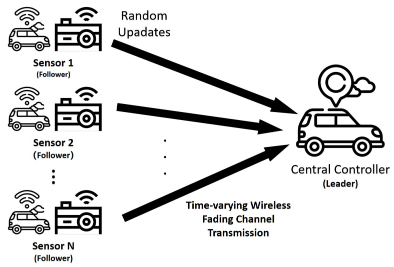

Fig. 1. Illustration of a vehicular network where a leader collects real-time information from followers through wireless channels.

denote the index of the current slot. We assume sensors are connected to the controller through different wireless time-varying channels with a shared bandwidth. At the beginning of each slot, at most M sensors can be scheduled to transmit their updates simultaneously. Let $u_n(t)$ denote whether sensor n is scheduled to transmit its update during slot t. If $u_n(t) = 1$, sensor n transmits its update to the controller; if $u_n(t) = 0$, sensor n idles in this slot. The bandwidth constraint implies the following restriction on $u_n(t)$:

$$\sum_{n=1}^{N} u_n(t) \leq M, \forall t. \tag{1}$$

To model the time-varying channels between sensors and the controller, we classify the channel condition into Q states. Let $q_n(t) \in \{1, 2, \ldots, Q\}$ be the current channel state between sensor n and the controller. Larger $q_n(t)$ demonstrates a noisier channel. We assume the channel state $q_n(t)$ appears independently with $\Pr(q_n(t) = q) = \eta_{n,q}$, and the distribution satisfies:

$$\sum_{q=1}^{Q} \eta_{n,q} = 1, \forall n = 1, \ldots, N. \tag{2}$$

To combat channel fading, each sensor consumes $w(q)$ units of energy when transmitting updates under channel state q. The energy consumption increases when the channel becomes noisier, so $w(q)$ is a non-decreasing function of q, i.e., $w(1) < \cdots < w(Q)$. Each sensor has a power constraint $\mathcal{E}_n$. The scheduling decision $\mathbf{u}_n(\pi) = [u_n(1), u_n(2), \ldots, u_n(T)]$ assigned to sensor n by a legal policy π must satisfy the power constraint, i.e.,

$$\lim_{T\to\infty} E_n(\mathbf{u}_n(\pi)) = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} u_n(t) w(q_n(t)) \leq \mathcal{E}_n, \forall n. \tag{3}$$

Different from [6], [7], we assume the transmission to be imperfect and each transmission under channel state q fails with probability $\varepsilon(q)$.

B. Age of Synchronization

In this work, we measure the freshness of data stored in the central controller using the metric *Age of Synchronization* (AoS) [1]. By definition, the AoS of sensor n is the time elapsed since information about sensor n in the controller becomes desynchronized compared with the sensor. To provide its closed-form expression, we first consider a single-sensor-discrete-time scenario.

Suppose that the i-th update of source n is generated in time slot g_i and is received by the controller by the end of slot r_i. Let $M(t) = \max_{i\in\mathbb{N}^+}\{i|r_i \leq t\}$ be the index of the freshest update received by the controller up to slot t. If the generation time-stamp of update $M(t)+1$ is earlier than time t, information at the central controller is desynchronized at time t. By definition, the AoS in slot t can be computed as follows:

$$x(t) = (t - g_{M(t)+1})^+, \tag{4}$$

where $(\cdot)^+ = \max\{0, \cdot\}$. Next, we will discuss how AoS of a sensor evolves with scheduling decision $u_n(t)$ and the external update according to (4).

Let $\Lambda_n(t) \in \{0, 1\}$ denote whether the n-th sensor observes a new update during slot t. When $\Lambda_n(t) = 1$, sensor n observes an update in slot t and we assume the sensor can transmit it after the beginning of slot $(t + 1)$. Suppose $\Lambda_n(t)$ follows i.i.d Bernoulli distribution with expectation $\mathbb{E}[\Lambda_n(t)] = \lambda_n$. In this work we assume the central controller is only interested in the "*freshest*" information from sensor n, so older update packets stored at sensor n will be discarded once a new packet arrives.

Denote $x_n(t)$ to be the AoS of sensor n at the beginning of slot t. Recall that $u_n(t)$ is the scheduling decision and $u_n(t) = 0$ implies sensor n is not scheduled in slot t. Then, if the current AoS $x_n(t) > 0$, which means information stored at the controller is already desynchronized, the AoS will increase linearly according to (4), i.e., $x_n(t+1) = x_n(t)+1$; otherwise, if $x_n(t) = 0$, the AoS of the next slot will be determined by whether the sensor generates a new update in slot t. If $\Lambda_n(t) = 1$, the information at the controller will be desynchronized at the start of next slot and $x_n(t + 1) = 1$; if $\Lambda_n(t) = 0$, the AoS will remain 0 at next slot, i.e., $x_n(t + 1) = 0$.

If $u_n(t) = 1$, the sensor will transmit a packet that is generated before slot t to the central controller in slot t. If currently $x_n(t) = 0$, then the AoS at the beginning of the next slot $x_n(t + 1)$ will be determined by whether there is an update in slot t: If $\Lambda_n(t) = 0$, the sensor does not observe a new update, then $x_n(t + 1) = 0$; otherwise, if $\Lambda_n(t) = 1$, the AoS becomes $x_n(t + 1) = 1$ due to desynchronization. If the current AoS $x_n(t) > 0$, information at the controller is already desynchronized. Then, the AoS in the next slot $x_n(t+1)$ will be determined by both transmission result (succeeds or fails) and $\Lambda_n(t)$: If the transmission is successful and $\Lambda_n(t) = 0$, the AoS will drop to 0, i.e., $x_n(t + 1) = 0$; if the transmission is successful but $\Lambda_n(t) = 1$, a new packet has been generated and the AoS will drop to 1, i.e., $x_n(t+1) = 1$; if the transmission fails, the AoS will increase, i.e. $x_n(t + 1) = x_n(t) + 1$. As a conclusion, we can formulate the following AoS evolution relationship:

$$x_n(t+1) = \begin{cases} 0, & x_n(t)=0, \Lambda_n(t)=0; \\ 1, & x_n(t)=0, \Lambda_n(t)=1; \\ 0, & \Lambda_n(t)=0, u_n(t)=1, \text{succeeds}; \\ 1, & \Lambda_n(t)=1, u_n(t)=1, \text{succeeds}; \\ x_n(t)+1, & \text{otherwise}. \end{cases} \tag{5}$$

C. Problem Formulation

We aim at designing a scheduling strategy that minimizes the expected average AoS performance of all N sensors over an infinite horizon. The average AoS of all sensors at the beginning of each slot following a specific scheduling policy π can be computed by:

$$J(\pi) = \lim_{T\to\infty} \{ \frac{1}{NT} \mathbb{E}_\pi \left[\sum_{t=1}^{T} \sum_{n=1}^{N} x_n(t) \right] \}. \tag{6}$$

Let Π_{NA} denote the class of all non-anticipating policies, where decisions $\{u_n(t)\}$ are made based on current and past information about AoS $\{x_n(\tau)\}_{\tau\le t}$ and channel states $\{q_n(t)\}$. No prediction on the future channel states can be used. We aim at designing a non-anticipating policy $\pi \in \Pi_{NA}$ for the central controller to schedule sensors under bandwidth and power constraints. The problem is formulated as follows:

Problem 1: (B&P-Constrained AoS)

$$\pi^* = \arg\min_{\pi\in\Pi_{NA}} \lim_{T\to\infty} \{ \frac{1}{NT} \mathbb{E}_\pi \left[\sum_{t=1}^{T} \sum_{n=1}^{N} x_n(t) \right] \}, \tag{7a}$$

$$\text{s.t.} \quad \mathbb{E}_\pi \left[\sum_{n=1}^{N} u_n(t) \right] \le M, \forall t, \tag{7b}$$

$$\lim_{T\to\infty} \frac{1}{T} \mathbb{E}_\pi \left[\sum_{t=1}^{T} u_n(t) w(q_n(t)) \right] \le \mathcal{E}_n, \forall n. \tag{7c}$$

III. Decoupled Single-Sensor Transmission Strategy

Problem 1 with the hard bandwidth is an NP-hard integer programming. Therefore, we relax the hard bandwidth (7b) and decouple *Problem* 1 into a single-sensor constrained Markov decision process (CMDP). By exploiting the threshold structure of its optimum solution, we solve the decoupled problem through linear programming (LP).

A. Single-Sensor Decouple

We first relax the hard bandwidth constraint (7b) into a time-average bandwidth constraint. After relaxation, more than M sensors can transmit updates in a single slot, but the average number of sensors scheduled per slot remains no larger than M. The relaxed problem is modified as follows:

Problem 2: (RB&P-Constrained AoS)

$$\pi_R^* = \arg\min_{\pi\in\Pi_{NA}} \lim_{T\to\infty} \{ \frac{1}{NT} \mathbb{E}_\pi \left[\sum_{t=1}^{T} \sum_{n=1}^{N} x_n(t) \right] \}, \tag{8a}$$

$$\text{s.t.} \quad \lim_{T\to\infty} \frac{1}{T} \mathbb{E}_\pi \left[\sum_{t=1}^{T} \sum_{n=1}^{N} u_n(t) \right] \le M, \tag{8b}$$

$$\lim_{T\to\infty} \frac{1}{T} \mathbb{E}_\pi \left[\sum_{t=1}^{T} u_n(t) w(q_n(t)) \right] \le \mathcal{E}_n, \forall n. \tag{8c}$$

To solve *Problem* 2, we place the relaxed bandwidth constraint (8b) into the object (7a) and formulate the Lagrange function as follows:

$$\mathcal{L}(\pi, W) = \lim_{T\to\infty} \frac{1}{NT} \mathbb{E}_\pi \left[\sum_{t=1}^{T} \sum_{n=1}^{N} [x_n(t) + W u_n(t)] \right] - \frac{WM}{N}. \tag{9}$$

Denote $\pi_R(W)$ to be the optimal policy that minimizes the Lagrange function $\mathcal{L}(\pi, W)$ with fixed Lagrange multiplier W under power constraint (8c). According to [14], the optimal policy π_R^* of *Problem* 2 is a mixture of no more than two such policies, denoted as $\pi_R(W_1)$ and $\pi_R(W_2)$, with different multipliers. Therefore, we will first focus on finding the optimal policy $\pi_R(W)$ with a fixed Lagrange multiplier W to minimize (9) and then talk about how to obtain W_1 and W_2 that constitute policy π_R^*.

Since the Lagrange function (9) has no bandwidth constraint, we can decouple *Problem* 2 into N single-sensor minimization problems with exclusive power constraints. The n-th single-sensor problem is explained as follows:

Problem 3: (Decoupled P-Constrained Cost)

$$\pi_n^* = \arg\min_{\pi\in\Pi_{NA}} \lim_{T\to\infty} \frac{1}{T} \mathbb{E}_\pi \left[\sum_{t=1}^{T} [x_n(t) + W u_n(t)] \right], \tag{10a}$$

$$\text{s.t.} \quad \lim_{T\to\infty} \frac{1}{T} \mathbb{E}_\pi \left[\sum_{t=1}^{T} u_n(t) w(q_n(t)) \right] \le \mathcal{E}_n. \tag{10b}$$

B. Constrained Markov Decision Process Formulation

When considering the decoupled model, we omit the subscript n for simplicity. Notice that *Problem* 3 can be formulated into a constrained Markov decision process (CMDP). The fundamental elements are explained as follows:

State Space: The state in slot t consists of the sensor's current AoS $x(t)$ and the channel state $q(t)$.

Action Space: The sensor's action of whether to transmit or idle in slot t is denoted as $a(t) \in \{0, 1\}$, where $a(t) = 0$ indicates the sensor is idle and $a(t) = 1$ implies a transmission decision.

Probability Transfer Function: Following the AoS evolution relationship (5), the probability transfer function from state (x, q) is as follows:

$$\Pr((0,q) \to (0,q') | a(t)=0) = (1-\lambda)\eta_{q'}, \tag{11a}$$

$$\Pr((0,q) \to (1,q') | a(t)=0) = \lambda\eta_{q'}, \tag{11b}$$

$$\Pr((0,q) \to (0,q') | a(t)=1) = (1-\lambda)\eta_{q'}, \tag{11c}$$

$$\Pr((0,q) \to (1,q') | a(t)=1) = \lambda\eta_{q'}, \tag{11d}$$

and when $x > 0$,

$$\Pr((x,q) \to (x+1,q') | a(t)=0) = \eta_{q'}, \tag{11e}$$

$$\Pr((x,q)\to(0,q')|a(t)=1)=(1-\lambda)(1-\varepsilon_q)\eta_{q'}, \quad (11f)$$

$$\Pr((x,q)\to(1,q)|a(t)=1)=\lambda(1-\varepsilon_q)\eta_{q'}, \quad (11g)$$

$$\Pr((x,q)\to(x+1,q)|a(t)=1)=\varepsilon_q\eta_{q'}. \quad (11h)$$

One-step Cost: For a given state (x,q), the one-step cost by taking action a is a sum of the current AoS and a scheduling penalty incurred by the Lagrange multiplier, i.e.,

$$C_X(x,q,a)=x+Wa. \quad (12)$$

The one-step power consumption is:

$$C_P(x,q,a)=\omega(q)a. \quad (13)$$

The object of the CMDP is to find the optimal scheduling policy under an average power constraint, which minimizes the overall cost containing both AoS and scheduling penalty:

$$\pi_M^*=\arg\min_{\pi\in\Pi_{NA}}\lim_{T\to\infty}\frac{1}{T}\mathbb{E}_\pi\left[\sum_{t=1}^{T}C_X(x,q,a)\right], \quad (14a)$$

$$\text{s.t.} \qquad \frac{1}{T}\mathbb{E}_\pi\left[\sum_{t=1}^{T}C_P(x,q,a)\right]\le\mathcal{E}. \quad (14b)$$

C. Characterization of the Optimal Policy

In this section, we explore the threshold structure of the optimal policy. Before we start, we provide definitions of stationary randomized policy and stationary deterministic policy:

Definition 1: Let Π_{SD} and Π_{SR} denote the class of stationary deterministic policy and stationary randomized policy respectively. Given the current state $(x(t)=x,q(t)=q)$, a stationary deterministic policy $\pi_{SD}\in\Pi_{SD}$ selects action $a(t)=1$ based on a deterministic mapping from state space to action space, i.e., $a(t):(x,q)\to\{0,1\}$. A stationary randomized policy $\pi_{SR}\in\Pi_{SR}$ selects action $a(t)=1$ with probability $\xi_{x,q}\in[0,1]$.

Similar to the analyses in [14] and [6], the optimal policy to the CMDP has the following property: The optimal policy π_M^* to the CMDP is a stationary randomized policy and it is a mixture of no more than two stationary deterministic policies π_{SD_1} and π_{SD_2}. Let $\lambda\in[0,1]$ denote the weight of following policy π_{SD_1}, then the optimal stationary randomized policy can be expressed as follows:

$$\pi_M^*=\lambda\pi_{SD_1}+(1-\lambda)\pi_{SD_2}. \quad (15)$$

The two stationary deterministic policies π_{SD_1} and π_{SD_2} can be obtained by solving the Lagrange problem of CMDP with power constraint. Let σ denote the Lagrange multiplier, then the Lagrange problem is explained as follows:

$$\pi_{SD}^*=\arg\min_{\pi\in\Pi_{NA}}\lim_{T\to\infty}\frac{1}{T}\mathbb{E}_\pi[\sum_{t=1}^{T}[C_X(x(t),q(t),a(t)) +\sigma C_P(x(t),q(t),a(t))]]-\sigma\mathcal{E}. \quad (16)$$

Denote γ to be the time average cost following the optimum policy π_{SD}^*. For fixed Lagrange multipliers W and σ, the optimum policy must satisfy the following Bellman equations where $V(x,q)$ is the cost-to-go function. By intuition, there is no need to send updates when the AoS is 0. Hence, we have:

$$V(0,q)+\gamma=\sum_{q'=1}^{Q}\eta_{q'}[\lambda V(1,q')+(1-\lambda)V(0,q')], \quad (17a)$$

and for $x>0$,

$$\begin{aligned}V(x,q)+\gamma=\min\{&C_X(x,q,0)+\sum_{q'=1}^{Q}\eta_{q'}V(x+1,q'),\\ &C_X(x,q,1)+\sigma C_P(x,q,1)+\sum_{q'=1}^{Q}\eta_{q'}[\varepsilon(q)V(x+1,q')\\ &+(1-\lambda)(1-\varepsilon(q))V(0,q')+\lambda(1-\varepsilon(q))V(1,q')]\}.\end{aligned} \quad (17b)$$

With the above Bellman equations, we can obtain the threshold structure of the optimum policy in the following lemma 1.

Lemma 1: For any channel state q, there exists a threshold τ_q so that it is always optimal to transmit when $x>\tau_q$ and always optimal to idle when $x<\tau_q$, i.e., $a^*(x,q)=1$ when $x>\tau_q$; $a^*(x,q)=0$ when $x<\tau_q$. Moreover, the set of τ_q is non-decreasing, i.e. $\tau_1\le\tau_2\le\cdots\le\tau_Q$.

Since the optimal stationary randomized policy to *Problem* 3 is a mixture of no more than two stationary deterministic policies with threshold structures, there exists a threshold X for policy π_M^* so that it is always optimal to transmit when AoS is larger than X.

D. Probabilistic Scheduling Policy for Single Sensor

We use a set of probabilities $\{\xi_{x,q}\}$ to denote a stationary randomized policy, where $\xi_{x,q}$ represents the probability of transmitting when AoS is x and channel state is q. Our goal is to find the optimal strategy $\{\xi_{x,q}^*\}$ that minimizes the overall cost. Due to the threshold structure, it is always optimal to transmit when $x>X$, hence $\xi_{x,q}^*=1,\forall x>X$. To compute $\{\xi_{x,q}^*\}_{x\le X}$, let μ_x be the steady state distribution that the AoS equals x and denote $y_{x,q}=\mu_x\eta_q\xi_{x,q}$. We then construct an LP as follows:

Theorem 1: The decoupled P-Constrained Problem 3 is equivalent to solving $\{\mu_x^*,y_{x,q}^*\}$ in the following LP problem:

$$\{\mu_x^*,y_{x,q}^*\}=\arg\min_{\{\mu_x,y_{x,q}\}}\left\{\sum_{x=1}^{X-1}x\mu_x+\frac{1}{1-\gamma}X\mu_X +\frac{\gamma}{(1-\gamma)^2}\mu_X+W\left(\sum_{x=1}^{X-1}\sum_{q=1}^{Q}y_{x,q}+\frac{1}{1-\gamma}\mu_X\right)\right\}, \quad (18a)$$

$$\text{s.t.}\quad \sum_{x=0}^{X-1}\mu_x+\frac{1}{1-\gamma}\mu_X=1, \quad (18b)$$

$$\lambda\mu_0=(1-\lambda)\left[\sum_{x=1}^{X-1}\sum_{q=1}^{Q}y_{x,q}(1-\varepsilon(q))+\mu_X\right], \quad (18c)$$

$$\mu_1=\lambda\left(\mu_0+\mu_X+\sum_{x=1}^{X-1}\sum_{q=1}^{Q}y_{x,q}(1-\varepsilon(q))\right), \quad (18d)$$

$$\mu_x = \mu_{x-1} - \sum_{q=1}^{Q}(1-\varepsilon(q))y_{x-1,q}, \tag{18e}$$

$$\sum_{x=1}^{X-1}\sum_{q=1}^{Q} y_{x,q}w(q) + \frac{1}{1-\gamma}\sum_{q=1}^{Q}\eta_q w(q)\mu_X \le \mathcal{E}, \tag{18f}$$

$$0 \le \mu_x \le 1, 0 \le y_{x,q} \le \mu_x\eta_q, \forall x, q. \tag{18g}$$

For a fixed multiplier W, let $\{\mu_x(W), y_{x,q}(W)\}$ be the solution of the LP problem and let $\{\xi_{x,q}(W)\}$ be the optimum policy. The threshold structure obtained in Section III-C implies the following properties on $\xi_{x,q}(W)$:

Theorem 2: For any channel state q and Lagrange multiplier W, there exists a threshold τ_q so that it is always optimal to transmit when $x > \tau_q$, i.e., $\xi_{x,q}(W) = 1$; it is always optimal to idle when $x < \tau_q$, i.e. $\xi_{x,q}(W) = 0$. The scheduling decision when $x = \tau_q$ is a randomized strategy with transmit probability $0 < \xi_{x,q}(W) \le 1$. Besides, the set of threshold τ_q is non-decreasing, i.e., $\tau_1 \le \tau_2 \le \cdots \le \tau_Q$.

The proportion of slots spent on scheduling the sensor, denoted as $S(W)$, can be computed as follows.

$$S(W) = \sum_{x=1}^{X-1}\sum_{q-1}^{Q} y_{x,q}(W) + \frac{1}{1-\gamma}\mu_X(W). \tag{19}$$

IV. Multi-Sensor Scheduling Policy

In this part, we first determine the Lagrange multiplier and find the optimal policy π_R^* to *Problem* 2 with a relaxed bandwidth constraint. We then propose an asymptotic optimal policy to solve the original multi-sensor scheduling problem with a hard bandwidth limit.

A. Multi-Sensor Scheduling with Relaxed Bandwidth

Denote $\{\mu_x^{n,W}, y_{x,q}^{n,W}\}$ to be the optimum solution to the LP of sensor n with Lagrange multiplier W. Suppose after the k-th iteration, the Lagrange multiplier is $W^{(k)}$. Using (19), the consumed bandwidth $b_n(W^{(k)})$ for each sensor n can be computed by:

$$b_n(W^{(k)}) = \sum_{x=1}^{X-1}\sum_{q=1}^{Q} y_{x,q}^{n,W^{(k)}} + \frac{1}{1-\gamma}\mu_X^{n,W^{(k)}}. \tag{20}$$

We start with $W^{(1)} = 0$. If $\sum_{n=1}^{N} b_n(W^{(1)}) \le M$, it indicates that the relaxed bandwidth can satisfy all sensors. In this case, the optimal distribution can be obtained from

$$\{\mu_x^{n,*}, y_{x,q}^{n,*}\} = \{\mu_x^{n,W^{(1)}}, y_{x,q}^{n,W^{(1)}}\}.$$

Otherwise, we obtain a Lagrange multiplier sequence $W^{(k)}$ iteratively through the subgradient method. The subgradient can be computed by:

$$dW^{(k)} = \sum_{n=1}^{N} b_n(W^{(k)}) - M. \tag{21}$$

Let $\delta^{(k)}$ be a sequence of decreasing stepsizes. The Lagrange multiplier used in the $(k+1)$-th iteration can be computed by:

$$W^{(k+1)} = W^{(k)} - \delta^{(k)} dW^{(k)}. \tag{22}$$

The iteration ends with $|W^{(k)} - W^{(k-1)}| < \epsilon$, then we choose two items from the obtained sequence:

$$W_u = \min_k\{W^{(k)} | \sum_{n=1}^{N} b_n(W^{(k)}) \ge M\}, \tag{23a}$$

$$W_l = \max_k\{W^{(k)} | \sum_{n=1}^{N} b_n(W^{(k)}) < M\}. \tag{23b}$$

The optimal stationary randomized policy π_R^* is a mixture of two optimum policies corresponding to multipliers W_u and W_l. The AoS and scheduling probability distribution to policy π_R^*, denoted by $\{\mu_x^{n,*}, y_{x,q}^{n,*}\}$, can be obtained by a weighted average of $\{\mu_x^{n,W_u}, y_{x,q}^{n,W_u}\}$ and $\{\mu_x^{n,W_l}, y_{x,q}^{n,W_l}\}$ as follows:

$$\begin{aligned}\{\mu_x^{n,*}, y_{x,q}^{n,*}\} =& \rho\{\mu_x^{n,W_u}, y_{x,q}^{n,W_u}\} \\ &+ (1-\rho)\{\mu_x^{n,W_l}, y_{x,q}^{n,W_l}\},\end{aligned} \tag{24}$$

where the weight λ is computed by:

$$\rho = \frac{M - \sum_{n=1}^{N} b_n(W_l)}{\sum_{n=1}^{N} b_n(W_u) - \sum_{n=1}^{N} b_n(W_l)}. \tag{25}$$

Finally, according to the threshold structure explained in Theorem 2, the optimum scheduling probability $\xi_{x,q}^{n,*}$ under relaxed bandwidth constraint can be computed by:

$$\xi_{x,q}^{n,*} = \begin{cases} \frac{y_{x,q}^{n,*}}{\mu_x^{n,*}\eta_{n,q}}, & x \le X; \\ 1, & x > X. \end{cases} \tag{26}$$

Let $\mathfrak{T}_\pi(t)$ be the set of sensors scheduled under a policy π at slot t. Policy π_R^* to *Problem* 2 is constructed as follows: At the beginning of each slot t, the central controller observes each sensor's AoS $x_n(t)$ and channel state $q_n(t)$, then select sensor n with probability $\xi_{x_n(t),q_n(t)}^{n,*}$ into set $\mathfrak{T}_{\pi_R^*}(t)$. The expected AoS performance of policy π_R^* formulates the lower bound of average AoS to *Problem* 1. Let π be a non-anticipated policy satisfying the hard bandwidth constraint in Problem 1, then we have:

$$J(\pi) \ge AoS_R^* = \frac{1}{N}\sum_{N=1}^{N}\sum_{x=1}^{X} x\mu_x^{n,*}. \tag{27}$$

B. Scheduling with Hard Bandwidth Constraint

In this part, we propose a truncated scheduling policy $\hat{\pi}$ to solve *Problem* 1 with a hard bandwidth based on π_R^*: recall that $\mathfrak{T}_{\pi_R^*}(t)$ is the set of sensors policy π_R^* chooses to schedule. The truncated scheduling policy $\hat{\pi}$ is constructed by randomly selecting as many but no more than M sensors in set $\mathfrak{T}_{\pi_R^*}(t)$ to transmit their updates in this time slot. It can be proved that our proposed policy approaches the AoS_R^* asymptoticly when the number of sensors $N \to \infty$.

V. Simulation

We first study the optimal transmit strategy in a single-sensor network with a power constraint. We consider a $Q = 4$ states channel with transmission power being $w_q = 2^q$. We assume each channel state appears with the same probability, i.e. $\eta_q = 0.25$, and the power constraint $\mathcal{E} = 0.45\sum_{q=1}^{Q}\eta_q w_q$.

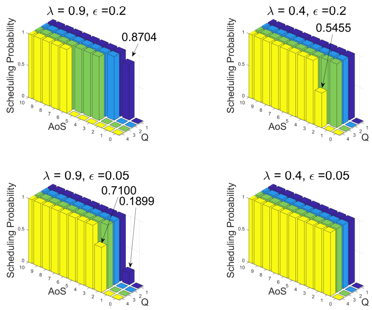

Fig. 2. Scheduling Strategy for Single Sensor with Power Constraint under Different Update Rates and Packet-loss Rates

The packet-loss probability $\varepsilon(q) = q\epsilon$. Fig. 2 plots the scheduling strategy under different packet-loss rates and different update rates. It is shown that with a lower update rate, the sensor updates less frequently and is more willing to transmit under lower AoS states in spite of bad channel states. Besides, with higher error rate ϵ, the sensor is more willing to sacrifice AoS performance to avoid transmitting in worse channels with larger q. The strategy verifies the threshold structure explained in Theorem 2.

Next, we provide simulation results in multi-sensor networks to demonstrate the average AoS performance of our proposed scheduling policy. We consider a $Q = 4$ state channel with distribution $\eta = [0.135, 0.239, 0.232, 0.394]$, and the packet-loss probability is $\varepsilon(q) = 0.1q$. Each sensor updates in every slot with probability $\lambda_n = 0.5$ and consumes $w_q = q$ energy to transmit one packet. We assume all sensors have the same power constraint $\mathcal{E} = 0.45 \sum_{q=1}^{Q} \eta_q w_q$. We simulate the scheduling process over a consecutive of $T = 10^6$ slots and study the average AoS performance as a function of number of sensors, $N = \{10, 15, \cdots, 50\}$. Denote $C_n(t)$ as the total power consumed by sensor n until time slot t and let $\mathfrak{R}(t) = \{n | \mathcal{E}_n t - C_n(t) \geq 0\}$ denote the set of sensors that have enough power to transmit in time slot t. We compare our proposed policy with a greedy policy which selects in each slot as many as no more than M sensors with the highest AoS from set $\mathfrak{R}(t)$ to send updates. As can be seen from Fig. 3, the proposed scheduling policy achieves a close average AoS performance to the lower bound and a near 50% AoS decrease compared to the greedy policy when $N = 50$.

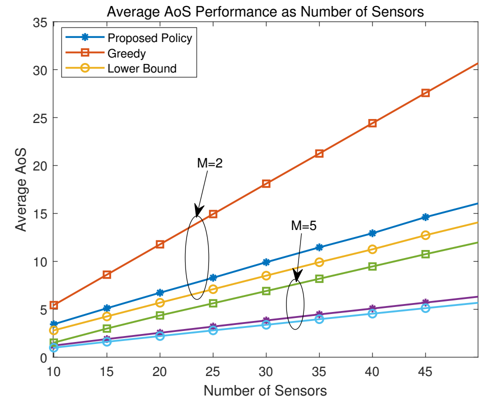

Fig. 3. Average AoS performance as a number of sensors N

VI. Conclusion

In this work, we consider a multi-sensor scheduling problem with bandwidth and power constraints in time-varying wireless channels. We measure the data freshness of each sensor through the metric *Age of Synchronization*. To minimize the average AoS performance, we first decouple the multi-sensor problem into a single-sensor CMDP, then reveal the threshold structure of the optimal transmission policy and finally approach the optimum solution through Linear Programming. We propose an asymptotic optimal policy to satisfy the hard bandwidth constraint. Our work suggests that sensors should exploit better channel states and send their updates at higher AoS in order to save power and raise transmission success probability, also allowing other sensors to transmit their more urgent updates.

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