

Scheduling to Minimize Age of Information in Multi-State Time-Varying Networks with Power Constraints

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Abstract—In this paper, we study how to collect fresh data in time-varying networks with power constrained users. We measure data freshness from the perspective of the central controller by using the metric *Age of Information*, namely the time elapsed since the generation time-stamp of the freshest information. We wonder what is the minimum AoI performance the network can achieve and how to design scheduling algorithms to approach it. To answer these questions when scheduling decisions are restricted to bandwidth constraint, we first decouple the multi-user scheduling problem into a single user constrained Markov decision process (CMDP) through relaxation of the hard bandwidth constraint. Next we exploit the threshold structure of the optimal policy for the decoupled single user CMDP and obtain the optimum solution through linear programming (LP). Finally, an asymptotic optimal truncated policy that can satisfy the hard bandwidth constraint is built upon the optimal solution to each of the decoupled single-user sub-problem. The performance is verified through simulations. Our investigation shows that to obtain a small AoI performance, the scheduler exploits good channels to schedule users supported by limited power. Users equipped with enough transmission power are updated in a timely manner such that the bandwidth constraint can be satisfied.

Index Terms—Age of Information, Cross-layer Design, Opportunistic Scheduling, Constrained Markov Decision Process

I. INTRODUCTION

Data freshness is gaining increasing importance in real-time services like real time positioning, monitoring and industrial control. To support these applications, users that track the corresponding physical phenomena are scheduled to send updates to the central controller via time-varying wireless channels. However, the bandwidth and interference constraint, the limited power resource of each user and the time varying nature of wireless channels create obstacles in scheduling strategy design. Moreover, traditional quality of service (QoS) guarantees such as latency and throughput have their limitations and may not guarantee a good data freshness performance. Thus, it is of great importance to revisit

sampling and scheduling strategies in wireless networks in order to obtain more fresh information.

A recently proposed metric, the *Age of Information* (AoI) [2], namely the time elapsed since the generation time-stamp of the freshest information stored at the receiver, has been widely adopted to measure data freshness in communication networks. Optimizing and analyzing AoI performance in point to point communication systems with power consumption constraint have been studied [3]–[9]. The optimal sampling and transmission strategy in the presence of queueing delay [5] and transmission failure [8] are shown to possess a threshold structure, i.e., sampling and update transmission occur when information at the receiver is no longer fresh while the update packets, if successfully received, can significantly reduce data staleness.

AoI performance and optimization in multi-user network have been studied in [10]–[18]. When all the users in the network are identical and update packets can be generated at will, a greedy policy that samples and schedules to transmit the user with the largest AoI is shown to be optimal [10]. When there is no packetloss in the network, this greedy policy is equivalent to the round robin strategy, which is shown to be order optimal when update packets can not be generated at will and arrive randomly [16]. In [11], it is revealed that users with relatively bad channel states are updated less frequently. Scheduling to minimize AoI performance in networks with time-varying channels are studied in [13], [14], where centralized and decentralized policies to minimize the average peak age of information (PAoI) are proposed. However, the channel model considered in these works have two states and no power adaptation strategy is used to combat wireless fading effect.

To combat the aforementioned fading effect, transmission power and bandwidth limitations, which appear at different layers of communication networks, cross-layer control strategies have been studied in [19]–[26] to minimize delay or maximize throughput. In [24], a Lazy scheduling policy that assigns scheduling decision based on the queue backlog is

Proof details are provided in our online report [1].

proposed. Considering time-varying fading nature of wireless channels, rate and power adaptation strategy is proposed in [25]. To minimize queueing delay in a point to point time-varying channel with average power constraint on the transmitter, a probabilistic scheduling strategy is proposed in [21], [22]. Although cross-layer strategies have been studied in delay minimization, throughput and utility maximization, the design to optimize Age of Information has not been very well studied.

To fill this gap, in our paper, we consider a single controller schedules multiple users to transmit updates in a wireless network. Similar to the cross-layer framework [27], the wireless link of each user is modeled to be multi-state time-varying and different level of transmission power is used in different channel state to guarantee success transmission. The overall objective is to minimize the expected average AoI performance when network is restricted to bandwidth constraint and scheduling decisions have to satisfy the power constraint of each user. Inspired by [26], we first relax the hard bandwidth constraint and decouple the multi-user scheduling problem into a single user constrained Markov decision process (CMDP). Then we propose a truncated scheduling policy that can achieve an asymptotic optimal average AoI performance over the entire network.

The remainder of this paper is organized as follows. The network model and the data freshness metric, AoI, are introduced in Section II. In Section III, we decouple the multi-user scheduling problem into single-user level CMDP and search for the optimal policy through LP. In Section IV, a truncated multi-user scheduling policy is proposed. Section V evaluates and analyzes the performance of the proposed algorithm and Section VI draws the conclusion.

Notations: Vectors and matrices are written in boldface lower and upper letters. The probability of event \mathcal{A} given condition \mathcal{B} is denoted as $\Pr(\mathcal{A}|\mathcal{B})$. The expectation operation with regard to random variable X is denoted as $\mathbb{E}_X[\cdot]$. The cardinality of set Ω is denoted as $|\Omega|$.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. Network Model

We consider a network with a central controller collecting time-sensitive data from N users via wireless links. Let the time be slotted, i.e., $t = \{1, \dots, T\}$. The central controller schedules users to transmit update at the beginning of each slot over time-varying fading channels. Let the indicator function $u_n(t)$ to be a scheduling decision. If $u_n(t) = 1$, then user n is scheduled to transmit update packet during slot t and the packet will be received successfully by the end of the slot. Due to the limited bandwidth resource, no more than M users can be scheduled simultaneously, which casts the following restrictions on $u_n(t)$:

$$\sum_{n=1}^N u_n(t) \leq M, \text{ for all } t. \quad (1)$$

We assume that the communication channel between the central controller and each user experiences an independent Q -state block fading, where Q is a positive integer. The channel state remains constant during a slot but follows an i.i.d fading process across the slots. Let $q_n(t) \in \{1, \dots, Q\}$ be a random variable that captures the channel state of user n during slot t , large $q_n(t)$ indicates that link n is more noisy and goes through stronger fading during slot t . Let the probability mass function of $q_n(t)$ be:

$$\Pr(q_n(t) = q) = \eta_{n,q}, \quad (2)$$

where $\eta_{n,q} \in [0, 1]$. For each user n , the sum of $\eta_{n,q}$ satisfies:

$$\sum_{q=1}^Q \eta_{n,q} = 1. \quad (3)$$

When user n is scheduled to transmit updates in a slot and the corresponding channel state is q , in order to guarantee successful transmission, it will consume $\omega(q)$ units of energy. To combat the effect of channel fading, more power will be consumed when the channel is more noisy, thus $\omega(1) < \dots < \omega(Q)$ is an increasing sequence. The transmitted packet will be successfully received by the central controller at the end of the slot. For a typical scheduling decision $\mathbf{u}_n(\pi) = [u_n(1), \dots, u_n(T)]$ related to user n , the average power consumed in T consecutive slots can be computed as follows:

$$E_n(\mathbf{u}_n(\pi)) = \frac{1}{T} \sum_{t=1}^T u_n(t) \omega(q_n(t)). \quad (4)$$

B. Age of Information

We measure data freshness of the central controller by using the metric *Age of Information* (AoI) [2]. By definition, the AoI is the time elapsed since the generation time-stamp of the freshest information at the receiver.

Let $x_n(t)$ be age of information of user n at the beginning of slot t . In this work, it is equivalent to the number of slots elapsed since the last delivery to user n . If $u_n(t) = 1$, fresh information about user n will be received by the central controller at the end of slot t , thus $x_n(t+1) = 1$; otherwise, since there is no update packet received from user n during slot t , $x_n(t)$ increases linearly and $x_n(t+1) = x_n(t) + 1$. The evolution of AoI $x_n(t)$ is organized as follows:

$$x_n(t+1) = \begin{cases} 1, & u_n(t) = 1; \\ x_n(t) + 1, & u_n(t) = 0. \end{cases} \quad (5)$$

C. Problem Formulation

For a given network setup with N users and channel states distributions $\{\eta_{n,q}\}$, we measure the data freshness of the entire network by following policy π in terms of the expected average AoI of all users at the beginning of each time slot for a consecutive of $T \rightarrow \infty$ slots, which is computed as follows:

$$J(\pi) = \lim_{T \rightarrow \infty} \left\{ \frac{1}{NT} \mathbb{E}_\pi \left[\sum_{t=1}^T \sum_{n=1}^N x_n(t) | \mathbf{x}(0) \right] \right\}, \quad (6)$$

where the vector $\mathbf{x}(t) = [x_1(t), x_2(t), \dots, x_N(t)] \in \mathbb{N}^N$ denotes the AoI of all users at the beginning of slot t . In this work, we assume that all the users have been synchronized initially, i.e., $\mathbf{x}(0) = \mathbf{1}$ and omit it henceforth.

Denote Π_{NA} to be the class of non-anticipated policies, i.e., scheduling decisions are made based on current and past AoI, channel states as well as their probability distributions, while no future information or prediction about channel states are exploited. The central controller is fully aware of the average power constraints of each user and aim at designing policy $\pi \in \Pi_{\text{NA}}$ in order to minimize the average expected AoI of the entire network. The bandwidth and power constrained AoI (B&P Constrained AoI) minimization problem is organized as follows:

Problem 1 (B&P-Constrained AoI):

$$\pi^* = \arg \min_{\pi \in \Pi_{\text{NA}}} \lim_{T \rightarrow \infty} \left\{ \frac{1}{NT} \mathbb{E}_{\pi} \left[\sum_{t=1}^T \sum_{n=1}^N x_n(t) \right] \right\}, \quad (7a)$$

$$\text{s.t. } \sum_{n=1}^N u_n(t) \leq M, \forall t, \quad (7b)$$

$$\lim_{T \rightarrow \infty} \frac{1}{T} \mathbb{E}_{\pi} \left[\sum_{t=1}^T u_n(t) \omega(q_n(t)) \right] \leq \mathcal{E}_n, \forall n. \quad (7c)$$

The hard bandwidth constraint (7b) in every slot t suggests that the *B&P-Constrained AoI* problem is an intractable integer programming problem. We tackle with this challenge through the following approaches:

- Inspired by [20], [26], [28], we relax the hard bandwidth constraint (7b) into a time-average constraint that allows more than M users to be scheduled simultaneously. Then the multi-user scheduling problem can be decoupled into single user CMDP.
- After solving the decoupled single user CMDP through LP in Sec. III-D, in Sec. IV, we propose a truncated scheduling policy that can satisfy the hard bandwidth constraint (7b).

III. SCHEDULING BY USER-LEVEL DECOMPOSITION

In this section, we start by relaxing and decoupling the *B&P-Constrained AoI* problem, then formulate the decoupled single user scheduling problem into a constrained Markov decision process (CMDP). We exploit the threshold structure of the optimal stationary randomized policy and the optimal solution is solved through linear programming (LP).

A. Single-User Level Decomposition

Let us first relax the hard constraint (7b) into a time-average constraint, the problem of scheduling multiple power constrained users with relaxed bandwidth constraint (RB&P-Constrained AoI) can be organized as follows:

Problem 2 (RB&P-Constrained AoI):

$$\pi_R^* = \arg \min_{\pi \in \Pi_{\text{NA}}} \lim_{T \rightarrow \infty} \left\{ \frac{1}{NT} \mathbb{E}_{\pi} \left[\sum_{t=1}^T \sum_{n=1}^N x_n(t) \right] \right\}, \quad (8a)$$

$$\text{s.t. } \frac{1}{T} \sum_{t=1}^T \sum_{n=1}^N u_n(t) \leq M, \quad (8b)$$

$$\lim_{T \rightarrow \infty} \frac{1}{T} \mathbb{E}_{\pi} \left[\sum_{t=1}^T u_n(t) \omega(q_n(t)) \right] \leq \mathcal{E}_n, \forall n. \quad (8c)$$

Next we establish the Lagrange function and place the relaxed constraint into the objective function (7a) as follows:

$$L(\pi, W) = \lim_{T \rightarrow \infty} \left\{ \frac{1}{NT} \mathbb{E}_{\pi} \left[\sum_{n=1}^N \sum_{t=1}^T \left(x_n(t) + W u_n(t) - \frac{WM}{N} \right) \right] \right\}. \quad (9)$$

For fixed W , the optimization problem (9) can then be decoupled into N single user cost minimization problem with average power consumption constraint. The objective of user n is to develop a scheduling strategy π_n such that under power constraint Eq. (7c), the average overall cost incurred by AoI and scheduling penalty can be minimized. The decoupled single user power constrained cost minimization problem is organized as follows:

Problem 3 (Decoupled P-Constrained Cost):

$$\pi_n^* = \arg \min_{\pi \in \Pi_{\text{NA}}} \mathcal{L}_n(\pi, W),$$

$$\text{where } \mathcal{L}_n(\pi, W) = \lim_{T \rightarrow \infty} \frac{1}{T} \mathbb{E}_{\pi} \left[\sum_{t=1}^T x_n(t) + W u_n(t) \right], \quad (10a)$$

$$\text{s.t. } \lim_{T \rightarrow \infty} \frac{1}{T} \mathbb{E}_{\pi} \left[\sum_{t=1}^T u_n(t) \omega(q_n(t)) \right] \leq \mathcal{E}_n. \quad (10b)$$

Since the primal relaxed problem (9) gets decoupled, we omit the subscript n henceforth.

B. Constrained Markov Decision Process Formulation

The decoupled single-user scheduling problem can be formulated into a CMDP that consists of a quadruplet $(\mathbb{S}, \mathbb{A}, \Pr(\cdot|\cdot), C(\cdot, \cdot))$, each item is explained as follows:

- **State Space:** The state of a user at the beginning of slot t is the current number of slots elapsed since the last update and the channel state $(x(t), q(t))$.
- **Action Space:** There are two possible actions $s \in \mathbb{A} = \{0, 1\}$, where $s(t) = 1$ denotes updates from the user is scheduled at the beginning of slot t , and $s(t) = 0$ represents that the user keeps idle and is not scheduled. Notice that $s(t)$ is different from scheduling decision $u(t)$, which has strict bandwidth constraint.
- **Probability Transfer Function:** The channel state $q(t)$ evolves independently of $x(t)$, hence according to Eq. (5), the probability transfer function from state (x, q) is organized as follows:

$$\Pr((x, q) \rightarrow (x+1, q')) = \eta_{q'}, \quad s = 0; \quad (11a)$$

$$\Pr((x, q) \rightarrow (1, q')) = \eta_{q'}, \quad s = 1. \quad (11b)$$

- **One-Step Cost:** For given state (x, q) , the one-step cost by taking action s contains AoI growth and scheduling penalty, which can be computed as follows:

$$C_X(x, q, s) = x + Ws, \quad (12a)$$

while the one-step power consumption is:

$$C_Q(x, q, s) = \omega(q)s. \quad (12b)$$

The objective of the decoupled CMDP problem is to design a scheduling policy π such that under the average power constraint, $\frac{1}{T}\mathbb{E}_\pi \left[\sum_{t=1}^T C_Q(x(t), q(t), s(t)) \right] \leq \mathcal{E}$, the overall cost containing both AoI and scheduling penalty over infinite horizon can be minimized, $\frac{1}{T}\mathbb{E}_\pi \left[\sum_{t=1}^T C_X(x(t), q(t), s(t)) \right]$.

C. Characterization of the Optimal Policy

In this part, we focus on exploiting the threshold structure of the optimal policy. First we provide the formal definition of a stationary randomized policy and stationary deterministic policy:

Definition 1: Let Π_{SR} and Π_{SD} denote the class of stationary randomized and stationary deterministic policy, respectively. Given observation $(x(t) = x, q(t) = q)$, a stationary randomized policy $\pi_{SR} \in \Pi_{SR}$ choose action $s(t) = 1$ with probability measure $\xi_{x,q} \in [0, 1]$ for all t . A stationary deterministic policy $\pi_{SD} \in \Pi_{SD}$ selects action $s(t) = a(x, q)$, where $a(\cdot) : (x, q) \rightarrow \{0, 1\}$ is a deterministic mapping from state space to action space.

According to [29, Theorem 4.4], the optimal policy to the above CMDP has the following property:

Corollary 1: An optimal stationary randomized policy $\pi^* \in \Pi_{SR}$ exists for the decoupled single user power constrained scheduling problem (3), and it is a mixture of no more than two stationary deterministic policies $\pi_{SD1}, \pi_{SD2} \in \Pi_{SD}$. Let λ be the weight of following stationary deterministic policy π_{SD1} and $1 - \lambda$ be the weight of following π_{SD2} . Then the optimum policy is:

$$\pi^* = \lambda\pi_{SD1} + (1 - \lambda)\pi_{SD2}. \quad (13)$$

Each of the deterministic policy can be obtained through the Lagrangian primal-dual method [29]. Let $\lambda \geq 0$ be the Lagrange multiplier related to the average power constraint, then the single user CMDP can be converted into an unconstrained MDP to minimize the following overall cost:

Problem 4 (Decoupled Unconstrained Cost):

$$\pi_{ud}^* = \arg \min_{\pi \in \Pi_{NA}} \lim_{T \rightarrow \infty} \frac{1}{T} \mathbb{E}_\pi \left[\sum_{t=1}^T (C_X(x(t), q(t), s(t)) + \lambda C_Q(x(t), q(t), s(t))) \right] - \lambda \mathcal{E}$$

For given Lagrange multiplier λ , a stationary deterministic policy to minimize the above unconstrained cost exists. Moreover, there exists a differential cost-to-go function $V(x, q)$ that satisfies the following Bellman equation:

$$V(x, q) + \gamma = \min \left\{ C_X(x, q, 0) + \sum_{q'=1}^Q \eta_{q'} V(x+1, q'), \right. \\ \left. C_X(x, q, 1) + \sum_{q'=1}^Q \eta_{q'} V(1, q') + \lambda C_Q(x, q, 1) \right\}, \quad (14)$$

where γ is the average cost by following the optimal policy. Next, we will prove the threshold structure of the stationary deterministic policy for given λ , which will present insight for the structure of the optimal stationary randomized policy to solve the CMDP problem (3).

Lemma 1: The optimal stationary deterministic policy for solving the *Decoupled Unconstrained Cost* minimization problem with fixed λ possesses a dual threshold structure, which is explained as follows:

- 1 For any channel state q , there exists a threshold τ_q , such that when $x \geq \tau_q$, the optimal action $s^*(x, q) = 1$ and when $x < \tau_q$, $s^*(x, q) = 0$.
- 2 The set of threshold is non-decreasing, i.e., $\tau_1 \leq \tau_2 \leq \dots \leq \tau_Q$.

D. Probabilistic Scheduling Policy for Single User Case

Denote $\xi_{x,q}$ to be the probability that the user is scheduled to send updates with age x and channel state q . We aim at finding a set of optimal transmission probability $\{\xi_{x,q}^*\}$ such that total cost of AoI performance and scheduling penalty for a single decoupled user can be minimized. From Sec. III(C), a stationary randomized policy that solves *Decoupled P-Constrained Cost* problem is a randomization between two stationary deterministic policies [29], each of them can be obtained by solving the *Decoupled Unconstrained Cost* minimization problem, which is an unconstrained MDP. Considering the threshold structure of them and Eq. (13), it can be concluded there exists set of non-decreasing thresholds τ_q , for each state (x, q) , if $x \geq \tau_q$, the stationary randomized policy is to schedule the user, i.e., $\xi_{x,q}^* = 1$. As an outcome, for each of the decoupled single user problem, when $x \geq \tau_Q$, the user will always be scheduled and the AoI x cannot be larger than the largest threshold τ_Q . To find the optimal policy, we choose a large X_{\max} that can guarantee $X_{\max} \geq \tau_Q$ in the following analysis.

Denote $\boldsymbol{\mu} = [\mu_1, \mu_2, \dots, \mu_{X_{\max}}]^T$ be the steady distribution of the user's AoI, where μ_x denotes the probability that $x(t) = x$. The probability transfer graph between the states is plotted in Fig. 1. Let α_x and β_x denote the one step state transition probability from $x(t) = x$ to $x(t+1) = x+1$ and from $x(t) = x$ to $x(t+1) = 1$, respectively, i.e.,

$$\alpha_x = \Pr(x(t+1) = x+1 | x(t) = x), \quad (15a)$$

$$\beta_x = \Pr(x(t+1) = 1 | x(t) = x). \quad (15b)$$

From the discussed threshold structure of deterministic policy, with properly selected X_{\max} , under the optimal scheduling policy, the steady state distribution $\mu_{X_{\max}}$ will be 0. And we have the following lemma:

Lemma 2: The forward state transfer probability α_x and β_x defined in (15a) and (15b) can be computed as follows:

$$\alpha_x = \sum_{q=1}^Q \eta_q (1 - \xi_{x,q}) \quad (16a)$$

$$\beta_x = \sum_{q=1}^Q \eta_q \xi_q. \quad (16b)$$

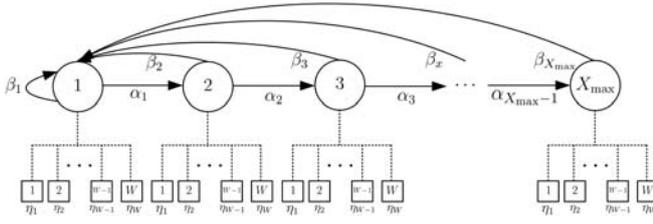


Fig. 1. Illustrative of the probability transfer graph for a stationary randomized policy. The circles denotes the AoI and the square denotes the channel states. The forward state transmission probability from AoI x to $x + 1$ is α_x and the backward state transmission probability from AoI x to 1 is β_x .

Let \mathbf{Q} be the probability transfer matrix between the states, which is,

$$\mathbf{Q} = \begin{bmatrix} \boldsymbol{\beta}^T \\ \mathbf{A}, \mathbf{0}_{X_{\max}-1} \end{bmatrix}, \quad (17)$$

where vector $\boldsymbol{\beta} = [\beta_1, \dots, \beta_{X_{\max}}]^T$ is the backward state transition probability vector and $\mathbf{A} = \text{diag}(\alpha_1, \dots, \alpha_{X_{\max}-1})$. Vector $\mathbf{0}_{X_{\max}-1}$ is a $(X_{\max} - 1)$ -dimension vector with all the elements being 0. According to property of the steady state distribution, we have $\mathbf{Q}\boldsymbol{\mu} = \boldsymbol{\mu}$. In addition, considering that $\mu_x = 0, \forall x \geq X_{\max}$, then we have $\sum_{x=1}^{X_{\max}} \mu_x = 1$. Thus, the steady distribution $\boldsymbol{\mu}$ relates to strategy $\{\xi_{x,q}\}$ is the solution to the following linear equations:

$$\begin{bmatrix} \mathbf{Q} - \mathbf{I}_{X_{\max}} \\ \mathbf{1}_{X_{\max}}^T \end{bmatrix} \boldsymbol{\mu} = \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix}, \quad (18)$$

where $\mathbf{1}_{X_{\max}}$ is a X_{\max} -dimension column vector with all the elements being 1.

Next, we will convert the search for the optimal stationary randomized scheduling strategy into an LP. We introduce a new set of variables $y_{x,q} = \mu_x \eta_q \xi_{x,q}$, each denotes the probability of the user is in state (x, q) and is scheduled to transmit an update. With this set of variables, we present the following theorem:

Theorem 1: The Decoupled P-Constrained Cost minimization problem is equivalent to the following LP problem:

$$\{y_{x,q}^*\} = \arg \min_{\{y_{x,q}\}, \{\mu_x\}} \left(\sum_{x=1}^{X_{\max}} \sum_{q=1}^Q W y_{x,q} + \sum_{x=1}^{X_{\max}} x \mu_x \right), \quad (19a)$$

$$\text{s.t. } \mu_1 = \sum_{x=1}^{X_{\max}} \sum_{q=1}^Q y_{x,q}, \quad (19b)$$

$$\mu_x = \mu_{x-1} - \sum_{q=1}^Q y_{x-1,q}, \quad (19c)$$

$$\sum_{x=1}^{X_{\max}} \mu_x = 1, \quad (19d)$$

$$y_{x,q} \leq \mu_x \eta_q, \quad (19e)$$

$$\sum_{x=1}^{X_{\max}} \sum_{q=1}^Q y_{x,q} \omega(q) \leq \mathcal{E} \quad (19f)$$

$$0 \leq \mu_x \leq 1, 0 \leq y_{x,q} \leq 1, \forall x, q. \quad (19g)$$

Till now, we construct an LP problem to obtain the optimum stationary randomized policy to minimize the total cost of a single user with fixed Lagrange multiplier W . Next, we can construct the optimal stationary randomized scheduling policy to minimize Lagrange function for a single user. According to the threshold structure of each deterministic policy, we will have the following properties on $\xi_{x,q}^*$:

Corollary 2: The set of optimal scheduling probabilities $\{\xi_{x,q}^*\}$ possesses the following characteristics:

- 1 For any channel state q :

$$\xi_{x_1,q}^* \leq \xi_{x_2,q}^*, \forall 1 \leq x_1 < x_2. \quad (20a)$$

- 2 For a specific AoI x :

$$\xi_{x,q_1}^* \geq \xi_{x,q_2}^*, \forall 1 \leq q_1 < q_2. \quad (20b)$$

With this corollary, we can then present the threshold structure of the stationary randomized policy:

Theorem 2: The optimal stationary randomized policy for solving the single-user scheduling problem (3) under power consumption constraint also possesses a threshold structure, which is explained as follows:

- 1 For any channel state q , there exists a threshold τ_q , such that when $x > \tau_q$, it is always optimal to schedule, i.e., $\xi_{x,q}^* = 1$ and when $x < \tau_q$, $\xi_{x,q}^* = 0$, while the scheduling decision at the τ_q may be a randomized strategy, i.e., $0 < \xi_{\tau_q,q}^* \leq 1$.
- 2 The set of threshold is non-decreasing, i.e., $\tau_1 \leq \tau_2 \leq \dots \leq \tau_Q$.

IV. MULTI-USER OPPORTUNISTIC SCHEDULING

In this section, we will provide an algorithm to determine the multiplier W such that relaxed bandwidth constraint can be satisfied. Then, we propose an asymptotic optimal truncated scheduling algorithm for the multi-user case that satisfies the original hard bandwidth constraint Eq. (7b).

A. Determination of Lagrange Multiplier

After solving the single user problem for fixed W , by combining the optimum scheduling strategy $s_n(t)$ for each of the user, the optimal policy $\pi_R^*(W)$ to minimize the Lagrange function Eq. (9) for fixed W can be obtained. Next, we describe how to obtain the optimal Lagrange multiplier W so that the *RB&P-Constrained AoI* problem can be solved.

Let $g(W)$ denote the Lagrange dual function, i.e.,

$$g(W) = \min_{\pi \in \Pi_{\text{NA}}} \mathcal{L}(\pi, W). \quad (21)$$

Since the relaxed problem gets decoupled into N single user CMDP, the dual function can be computed by:

$$g(W) = \frac{1}{N} \sum_{n=1}^N g_n(W) - WM, \text{ where} \quad (22)$$

$$g_n(W) = \min_{\pi_n \in \Pi_{\text{NA}}} (\mathcal{L}_n(\pi_n, W)), \text{ s.t. Eqn. (7c).}$$

Notice that by Theorem 1, the CMDP that minimizes $\mathcal{L}_n(\pi, W)$ is equivalent to an LP, then the duality gap between $g_n(W)$ and the CMDP is zero [29]. Let $\bar{X}_n(W)$ and $\bar{A}_n(W)$ denote the average AoI and the average activation probability for user n , respectively. By computing the optimum resource allocation vector $\{y_{x,q}^*\}$ through solving LP (19), the dual function $g_n(W)$ can be computed as follows:

$$g_n(W) = \bar{X}_n(W) + W\bar{A}_n(W) \quad (23a)$$

$$\text{where } \bar{X}_n(W) = \sum_{x=1}^{X_{\max}} x\mu_x^*, \quad (23b)$$

$$\bar{A}_n(W) = \sum_{x=1}^{X_{\max}} \sum_{q=1}^Q y_{x,q}^*. \quad (23c)$$

Finally, we apply the subgradient descent method to search for the Lagrange optimizer. Let $W^{(k)}$ be the Lagrange multiplier used in the k^{th} iteration. According to [30, Eq. 6.1.1], the subgradient at $W^{(k)}$ can be computed by:

$$d_W g(W^{(k)}) = \sum_{n=1}^N \bar{A}_n(W^{(k)}) - M. \quad (24)$$

We start with $W^{(0)} = 0$, if $\sum_{n=1}^N \bar{A}_n(W^{(0)}) - M \leq 0$, then scheduling does not have to consider the relaxed bandwidth constraint. Otherwise, we adopt an iterative algorithm update. By choosing a set of stepsizes γ_k similar to [8], the multiplier for the next iteration can be computed by:

$$W^{(k+1)} = W^{(k)} + \gamma_k d_W g(W^{(k)}). \quad (25)$$

The iteration ends until $|W^{(k)} - W^{(k+1)}| < \varepsilon$.

However, since the *RB&P-Constrained AoI* is a constrained Markov decision process, the optimum scheduling policy of which should be a randomization between no more than two policies, each is the solution to minimize the Lagrange function Eq. (9). The randomization between the two policies will enable us to satisfy the relaxed bandwidth constraint Eq. (8b) in the *RB&P-Constrained AoI*. Next, we will talk about how to obtain the optimum randomized strategy from the obtained Lagrange multipliers sequence $\{W^{(k)}\}$.

Let W_l and W_u be two Lagrange multipliers chosen from sequence $W^{(k)}$,

$$W_l = \arg \max_{W^{(k)}} \sum_{n=1}^N \bar{A}_n(W^{(k)}), \text{ s.t. } \sum_{n=1}^N \bar{A}_n(W^{(k)}) \leq M, \quad (26a)$$

$$W_u = \arg \min_{W^{(k)}} \sum_{n=1}^N \bar{A}_n(W^{(k)}), \text{ s.t. } \sum_{n=1}^N \bar{A}_n(W^{(k)}) \geq M. \quad (26b)$$

Then, let M_l and M_u be the total bandwidth used with respect to minimize the function Eq. (9). Let $\{\mu^{n,l}, \mathbf{y}^{n,l}\}$ be solution to user n 's LP problem (19) with multiplier W_l and $\{\mu^{n,u}, \mathbf{y}^{n,u}\}$ is the solution with multiplier W_u . To satisfy the relaxed bandwidth constraint, the optimum distribution $\{\mu^{n,*}, \mathbf{y}^{n,*}\}$ of the relaxed problem is a linear combination

of $\{\mu^{n,l}, \mathbf{y}^{n,l}\}$ and $\{\mu^{n,r}, \mathbf{y}^{n,r}\}$, which can be computed as follows:

$$\{\mu^{n,*}, \mathbf{y}^{n,*}\} = \lambda \{\mu^{n,l}, \mathbf{y}^{n,l}\} + (1 - \lambda) \{\mu^{n,u}, \mathbf{y}^{n,u}\}, \quad (27)$$

where the coefficient λ can be computed in a similar manner to [8]:

$$\lambda = \frac{M_u - M}{M_u - M_r}.$$

Notice that $\{\mu^{n,*}, \mathbf{y}^{n,*}\}$ still satisfy the constraint of the LP problem for user n . Consider the structure of each *Decoupled P-Constrained Cost* problem, the optimum scheduling strategy π_R^* for the *RB&P-Constrained* is then constructed as follows:

In each slot t , the central controller observes the current AoI $x_n(t)$ and channel state $q_n(t)$ of user n , a scheduling decision $s_n(t) = 1$ is then made with probability $\xi_{x_n(t), q_n(t)}^{n,*}$ is can be computed as follows:

$$\xi_{x,q}^n = \begin{cases} 1, & \xi_{x-1,q}^n = 1 \text{ or } \mu_x^{n,*} = 0 \text{ or } x \geq X_{\max}; \\ \frac{y_{x,q}^{n,*}}{\mu_x^{n,*} \eta_q}, & \text{otherwise.} \end{cases} \quad (28)$$

Finally, the minimum AoI performance to the *RB&P-Constrained AoI* problem can be computed through according to the optimizer $\{\mu^{n,*}, \mathbf{y}^{n,*}\}$, which also formulates the lower bound on the AoI performance to the primal *B&P-Constrained AoI*:

$$\text{AoI}_{\text{LB}} = \text{AoI}_{\text{R}}^* = \sum_{n=1}^N \sum_{x=1}^{X_{\max}} x \mu_x^{n,*}. \quad (29)$$

B. Multi-User Opportunistic Scheduling with Hard Constraint

In this part we construct a truncated policy π based on optimal scheduling policy for each of the decoupled user and solve the primal *B&P-Constrained AoI* problem. Let π_R^* be the optimum scheduling policy obtained in Sec. IV(A), where $s_n(t)$ is the scheduling decision under the relaxed constraint, which measures if user n is eager be scheduled. Denote $\Omega(t) = \{n | s_n(t) = 1\}$ as the set of users that are eager to be scheduled. The scheduling decision $u_n(t)$ under hard bandwidth constraint is then carried out as follows:

- If $|\Omega(t)| \leq M$, i.e., the available bandwidth can satisfy all the users that are eager to send updates, then all the users that are eager to be scheduled can send their updates, i.e., $u_n(t) = 1, \forall s_n(t) = 1$.
- Otherwise if $|\Omega(t)| > M$, the central controller selects a subset of $\mathcal{M}(t) = |M| \in \Omega(t)$ users randomly from $\Omega(t)$ and schedule them to send updates. Those users that are in set $\Omega(t)$ but not selected in $\mathcal{M}(t)$ is not selected because of limited bandwidth constraint.

Theorem 3: With the proportion of scheduling resources $\frac{M}{N} = \theta$ keeps a constant, the deviation from the optimal scheduling policy for a network with N users under the proposed truncated policy $\tilde{\pi}$ is $\mathcal{O}(\frac{1}{\sqrt{N}})$. Thus, with $N \rightarrow \infty$ and $\frac{M}{N} = \theta$, the proposed truncated policy is shown to be asymptotically optimal for the primal *B&P-Constrained AoI* problem with hard bandwidth constraint.

V. SIMULATIONS

In this section, we provide simulation results to demonstrate the performance of the proposed scheduling policy. From [11], the optimal policy to minimize AoI performance when all the users are identical is a greedy policy that selects the user with the largest AoI. If there is no packet loss in the network, the greedy policy is equivalent to round robin, which requires a minimum power consumption of $\mathcal{E}_n^{\text{RR}} = \frac{M}{N} \sum_{q=1}^Q \eta_{m,q} \omega(q)$ for user n . We measure power consumption constraint through ratio $\rho_n = \mathcal{E}_n / \mathcal{E}_n^{\text{RR}}$. We consider a $Q = 4$ states time-varying channel, the distribution is assumed to be $\eta = [0.135, 0.239, 0.232, 0.394]$ and $\omega(q) = q$ for all users. Simulation results are obtained over a consecutive of $T = 10^6$ slots.

Fig. 2 studies average AoI performance as a number of users, $N = \{10, 15, \dots, 50\}$. The power constraint factor is taken from $[0.2, 1.6]$, i.e., $\rho_n = 0.2 + \frac{1.4}{N}(n-1)$ and the bandwidth $M = \{2, 5\}$. Denote $C_n(t)$ as the total power consumed by user n until slot t and let $\mathcal{R}(t) = \{n | \mathcal{E}_n t - C_n(t) \geq 0\}$ be the set of users that has enough power to support transmission in slot t . We compare the proposed policy with a naive greedy policy that selects no more than M users with the largest AoI from set $\mathcal{R}(t)$ for scheduling. As can be seen from the figure, the proposed truncated scheduling achieves a close average AoI performance to the lower bound and can achieve more than 30% AoI decrease compared to the greedy algorithm when $N = 50$.

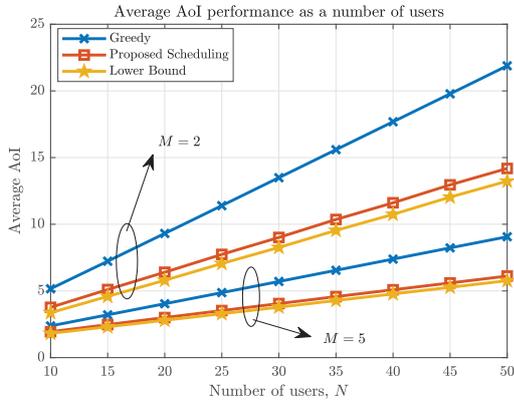


Fig. 2. Average AoI performance as a number of users N .

Fig. 3 studies the asymptotic average AoI performance as a number of users, with $\frac{M}{N} = \{\frac{1}{5}, \frac{1}{9}\}$. The power constraint of each user is selected by $\rho_n = 0.2 + \frac{1.4}{N}(n-1)$. As can be observed from the figure, the difference between the proposed strategy and the lower bound decreases with N . The asymptotic performance is also verified in simulation results.

We visualize the scheduling policy for some representative users in Fig. 4. The network consists of $N = 8$ users with bandwidth $M = 2$, the power constraint factor for each user is $\rho_n = 0.2n$. Fig. (a)-(d) demonstrate user $\{1, 2, 7, 8\}$ with power constraint $\rho = \{0.2, 0.4, 1.4, 1.6\}$, respectively. In Fig. 4(a)-(b), the transmission power for each user is

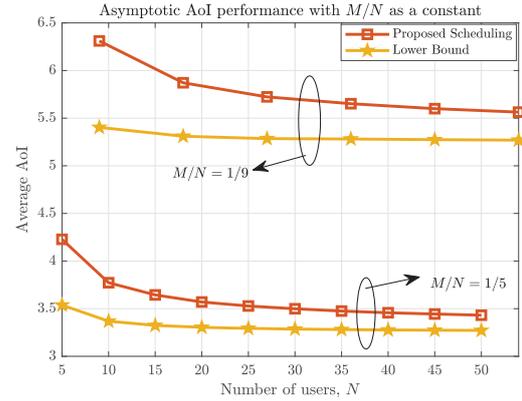


Fig. 3. Asymptotic average AoI performance as a number of users N , available bandwidth is chosen by $M/N = \{\frac{1}{5}, \frac{1}{9}\}$.

limited, the scheduling threshold τ_q is an increasing sequence as channel state q . Moreover, the threshold of each channel stated in Fig. 4(b) is smaller than corresponding threshold in Fig. 4(a), indicating that transmission is more likely to happen as a result of more available transmission power. The optimal strategy for a single user seeks to exploit a good channel state in order to satisfy the power constraint, while trying to keep the AoI small. If unfortunately the channel state is always bad, he will keep waiting until data staleness cannot be bare anymore or the channel state turns good. By comparing Fig. 4(a)-(b), the scheduler tries to make full use of the transmission power through a refinement of activation thresholds. When $\rho = 1.4$, power consumption is not a problem, all the channel states shares identical activation threshold that can satisfy the relaxed bandwidth constraint. Considering the greedy AoI performance depicted in Fig. 2, although greedy algorithm attempts to use up the power and bring smaller AoI performance to users equipped with enough power, it fails to exploit a good channel and opportunistically schedules those power constrained users, hence lead to a much higher AoI performance. Thus, for a network with different power constrained users, the scheduler seeks good channels to carry out scheduling decisions for those power constrained users, while users supported by enough power are updated in a timely manner that can satisfy bandwidth constraint.

VI. CONCLUSIONS

In this work, we investigate into the problem of age minimization scheduling in power constrained wireless networks, where communication channels are multi-state time varying and different levels of transmission power is adopted to ensure successful transmission. We decouple the multi-user scheduling problem into a single user level constrained Markov decision process. We reveal the threshold structure of the optimal stationary randomized policy for the single user and convert the optimal scheduling problem into a linear programming. An asymptotic optimal truncated scheduling policy for multi-user scenario that satisfies the hard bandwidth constraint is proposed. It is revealed that when power

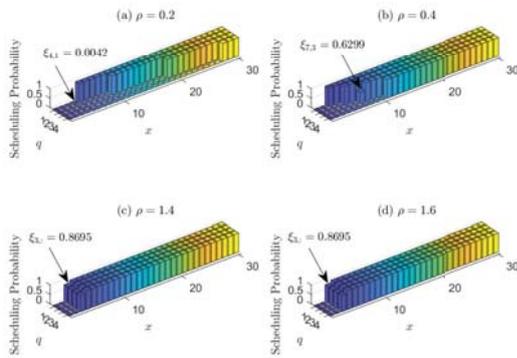


Fig. 4. Scheduling decisions for users with different power constraint ρ_n .

of the user is very limited, the scheduler seeks to exploit a good channel state while keeping the information fresh and minimize the scheduling opportunities. Users equipped with sufficient power are updated in a timely manner that can satisfy the hard bandwidth constraint.

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