Early Drop: A Packet-Dropping Incentive Rate Control Mechanism to Keep Data Fresh under Heterogeneous QoS Requirements

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***Abstract*—In this paper, we consider a distributed rate control problem with two selfish users. One user focuses on optimizing the data freshness while the other one aims at optimizing the ”power” of throughput and the transmission delay. Two users aim at achieving their own goals by controlling the source sending rate. We formulate the problem as an uncooperative rate control game with two users and show that the Nash Equilibrium (NE) of the game is inefficient compared to the global optimum solution. To improve the efficiency, we propose a linear packet-dropping incentives in the uncooperative rate control game with two users, so that users are encouraged to control their rates near the global optimum sending rates. We show that the proposed incentives can improve the Price of Anarchy (PoA) performance.**

***Index Terms*—Age of Information, Rate Control**

I. Introduction

Providing low latency, high throughput and fresh data to receivers is a crucial task for broadband broadcast networks. Due to the heterogeneous demands from the multi-users sharing the networks and the distributed rate control nature of broadband networks, it is crucial for the base station or the network gateways of the broadcast networks to design proper incentives to ensure network fairness so that the heterogeneous requests can be satisfied.

Recently, broadcast networks have been providing increasingly interacting services to network users, and the *freshness* of the service data greatly affects user experiences. To quantify how ”fresh” the data is at the receiver, the Age of Information (AoI), namely the amount of time elapsed since the current up-to-date sample at the receiver is generated, has recently been proposed [1]. A good data freshness performance requires the communication system to possess both high throughput and low delay, thus it is crucial for the transmitter to select an appropriate transmission rate, so that the trade-off between communication latency and system throughput can be achieved.

Scheduling and rate control strategies to optimize data freshness performance have been widely studied in [2]–[4]. It is revealed that users with relatively high AoI and better channel state should be scheduled with higher priority in order to guarantee a good AoI performance over the entire network. However, the communication networks are shared by users with heterogeneous requests (e.g., total throughput, transmission delay). Scheduling algorithms should take these heterogeneous demands into account. When half of the users have an average timely throughput constraint, [5] proposed algorithm based on Constrained Markov Decision Process (CMDP). Notice that the above algorithm is implemented in a centralized manner. When users with throughput and freshness demands coexist in a network, distributed rate selection has been studied in [6]. It is found that with no pricing or congestion control incentives, users who care more about throughput will take more active transmitting strategies and thus occupy most of the bandwidth of the network. This phenomena will cause Quality of Service (QoS) unfairness between the multiple users. Although packet-loss incentives have been proposed to guarantee the utility ”fairness” in multi-user queueing networks, mechanism designed to promote fairness between AoI optimal user and throughput optimal user requires further investigation.

To fill this gap, we studied distributed rate control problem in the co-existence of an AoI optimal user and an throughput optimal user in queueing networks with a shared server. The selfish behaviours of the two users will cause the unfairness and lead to the Price of Anarchy (PoA) performance of the system goes to infinity. To overcome this issue, we study and propose a packet-drop incentive to promote fairness. When the throughput optimal user tends encourage the corresponding source to send at a higher transmission rate, the packet-dropping incentive penalizes this behaviour by dropping more packets and encourages the throughput optimal user to send at a lower rate, so that the AoI performance can be kept small. The performance of the algorithm is evaluated via simulations.

The rest of the paper is organized as follows. The system model is introduced and the uncooperative rate control games are formulated in Section II. Section III compares the Nash Equilibrium (NE) and the global optimal point of the game. Packet-dropping incentives are proposed and analyzed in Sec-

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tion IV and Section V draws the conclusion.

II. Problem Formulation

Consider two sources sending information flows to two receivers through a FCFS server as depicted in Fig. 1. Each source i submits data packets into the system with independent Poisson distribution parameterized by λ_i and the service time for each packet follows an exponential distribution with parameter μ. Assume that user 1 wants the latest information

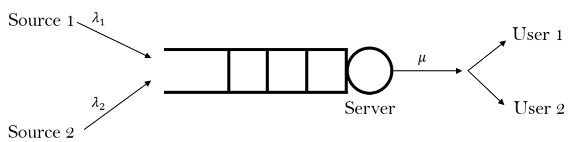

Fig. 1. System Model

about source 1 and thus source 1 samples and submits the corresponding up-to-date packets to the system. We use *Age of Information* [1] to measure the data freshness of the information stored at user 1. By definition, the AoI of user 1 at time t, denoted by $x_1(t)$ is the time elapsed since freshest information at the receiver is generated. The inter-sampling interval follows an exponential distribution with parameter λ_1. Let $A(\lambda_1, \lambda_2)$ be the average Peak AoI (PAoI) when source 1 and source 2 submit packets with rate λ_1 and λ_2, respectively. According to [7], $A(\lambda_1, \lambda_2)$ can be computed by:

$$A(\lambda_1, \lambda_2) = \frac{1}{\mu}\left(\frac{1}{\rho_1} + \frac{1}{(1-\rho)}\right), \tag{1}$$

where $\rho_i = \frac{\lambda_i}{\mu}$ and $\rho = \frac{\lambda_1+\lambda_2}{\mu}$.

Assume that user 2 wants to maximize the total throughput while keeping a small average per packet transmission delay. Such requirement can be characterized by the "*power*" defined in [8], i.e.,

$$\text{Power} = \frac{\text{Throughput}^{\alpha}}{\text{Delay}},$$

where α is chosen based on the relative emphasis of throughput versus delay. Here $\alpha > 1$ indicates throughput is more important than delay and $\alpha < 1$ suggests that a small delay is more important than throughput. Notice that the average transmission delay of each packet is $\frac{1}{\mu}\frac{1}{1-\rho}$. Let $P(\lambda_1, \lambda_2)$ be the "power" of user 2 when user 1 and user 2 send packets with rate λ_1 and λ_2 respectively, i.e.,

$$P(\lambda_1, \lambda_2) = \frac{\lambda_2^{\alpha}}{d} = \mu{\lambda_2}^{\alpha}(1-\rho). \tag{2}$$

In this work we consider that each source i attempts to optimize its own quality of service (minimize his own AoI or maximize his "power" performance) by controlling his own sending rate λ_i (or equivalently ρ_i). In this paper, we assume the queue to be stable, i.e., $0 < \rho_1 + \rho_2 < 1$. In Section III, we form the aforementioned distributed rate control problem as a non-cooperative game and show that the selfishness nature of each user may lead to a high PAoI performance of user 1. To alleviate this problem, in Section IV, we propose a packet-drop incentive approach that alleviate the selfish behaviour of user 2.

III. Non Cooperative Game with No Packet-loss Incentives

We first consider both users aim to reach their goal (minimize AoI of maximize power) by controlling their sending rate with knowledge of the system service rate μ. To facilitate further analysis, each user control its own rate λ_i by adjusting the corresponding system utilization ratio $\rho_i = \lambda_i/\mu$. Due to their selfish nature, source 1 aims at minimizing the AoI of user 1 without caring the throughput performance of user 2. Let the utility function of user 1 be the average Peak Age of Information (Ignoring the constant coefficient), i.e., $U_1(\rho_1, \rho_1) \propto A(\lambda_1, \lambda_2)$, the optimization problem can be formulated as:

Problem 1 (Opt-Source 1 without packet-dropping mechanism):

$$\min_{\rho_1} U_1(\rho_1, \rho_2) = \frac{1}{\rho_1} + \frac{1}{1-\rho_1-\rho_2}, \text{s.t., } \rho_1 \in [0, 1]. \tag{3}$$

While source 2 aims at maximizing the "power" of user 2 by controlling ρ_2 without caring the PAoI performance of user 1. To avoid the trivial case when investigating the social welfare optimization case when $\alpha \to \infty$, in this case we define the utility function and formulate the optimization problem of user 2 as follows:

Problem 2 (Opt-Source 2 without packet-dropping mechanism):

$$\max_{\rho_2} U_2(\rho_1, \rho_2) = \begin{cases} \rho_2^{\alpha}(1-\rho_1-\rho_2), & \alpha < 1; \\ \rho_2(1-\rho_1-\rho_2)^{1/\alpha}, & \alpha \geq 1. \end{cases} \tag{4}$$

s.t., $\rho_2 \in [0, 1]$.

Remark 1: This definition is in accordance to "power" from the perspective of the user. When $\alpha < 1$, delay is more important than throughput for user 2 and when $\alpha \geq 1$ throughput is more important.

Definition 1: A system utilization ratio allocation pair (ρ_1^*, ρ_2^*) is a Nash Equilibrium (NE) point for the above two user non-cooperative gave if and only if:

(1). When source 2 selects system ratio ρ_2^*, user 1 cannot achieve a smaller value of U_1 than $U_1(\rho_1^*, \rho_2^*)$ by changing ρ_1, i.e., $U_1(\rho_1^*, \rho_2^*) \leq U_1(\rho_1, \rho_2^*), \forall \rho_1 \in (0, 1-\rho_2^*)$.

(2). When source 1 selects system ratio ρ_1^*, user 2 cannot achieve a larger value of U_2 than $U_2(\rho_1^*, \rho_2^*)$, i.e., $U_2(\rho_1^*, \rho_2^*) \geq U_2(\rho_1^*, \rho_2), \forall \rho_2 \in (0, 1-\rho_1^*)$.

Proposition 1: The above two sources non-cooperative rate control problem can be formulated as a game and a unique NE system utilization ratio allocation pair (ρ_1^*, ρ_2^*) can be computed by:

$$(\rho_1^*, \rho_2^*) = \left(\frac{1}{\alpha+2}, \frac{\alpha}{\alpha+2}\right). \tag{5}$$

Notice that when $\alpha \to \infty$, indicating user 2 is more concerned with throughput, source 2 will try to occupy the system by submitting packets at a higher rate with $\rho_2 \to 1$.

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As a result, source 1 will try to achieve a trade-off between transmission delay and throughput by decreasing ρ_1 even when source 2 is trying to occupy the whole bandwidth. This is obviously not fair. Next, we will show the inefficiency of the non-cooperative game through the Price of Anarchy (PoA).

A. Social Welfare Optimization

Since user 1 wants to minimize the PAoI while user 2 wants to maximize the "power", to ensure efficiency with fairness, a common way to define the system total utilization comes to be:

$$U_{\text{tot}}(\rho_1,\rho_2) = -\log U_1(\rho_1,\rho_2) + \log U_2(\rho_1,\rho_2). \tag{6}$$

Then the social welfare optimization problem can be formulated as follows:

$$(\rho_1^{**},\rho_2^{**}) = \arg\max_{\rho_1,\rho_2} U_{\text{tot}}(\rho_1,\rho_2), \tag{7a}$$

$$\text{s.t. } \rho_1+\rho_2 \le 1, \rho_1,\rho_2 \in [0,1]. \tag{7b}$$

By forcing $\frac{\partial U_{\text{tot}}(\rho_1,\rho_2)}{\partial \rho_i} = 0 (i=1,2)$, we can get:

Proposition 2: The optimum solution for the social welfare optimization problem is:

$$(\rho_1^{**},\rho_2^{**}) = \begin{cases} \left(\frac{2}{3(\alpha+2)}, \frac{\alpha}{\alpha+2}\right), \alpha<1; \\ \left(\frac{\alpha^2+\alpha}{(2\alpha+1)^2}, \frac{\alpha}{2\alpha+1}\right), \alpha \ge 1. \end{cases} \tag{8}$$

B. Price of Anarchy and Price of Stability Analysis

To evaluate the efficiency of the two-user game, we calculate the price of anarchy (PoA) and price of stability (PoS), which are widely used in game theory. By definition PoA and PoS can be calculated as follows:

$$\text{PoA} \triangleq \max_{(\rho_1,\rho_2)\in\mathcal{E}} \frac{\exp\left(U_{\text{tot}}^{\text{OPT}}\right)}{\exp\left(U_{\text{tot}}^{\text{NE}}(\rho_1,\rho_2)\right)}, \tag{9}$$

$$\text{PoS} \triangleq \min_{(\rho_1,\rho_2)\in\mathcal{E}} \frac{\exp\left(U_{\text{tot}}^{\text{OPT}}\right)}{\exp\left(U_{\text{tot}}^{\text{NE}}(\rho_1,\rho_2)\right)}, \tag{10}$$

where $\mathcal{E}$ is the set of the Nash Equilibrium points. As shown in Proposition 1, for the above two user non-cooperative rate control game with no incentives, there exists only one NE point which we have noted as (ρ_1^*,ρ_2^*) before, hence PoA=PoS and in the following analysis we denote

$$\text{PoA} = \frac{\exp\left(U_{\text{tot}}(\rho_1^{**},\rho_2^{**})\right)}{\exp\left(U_{\text{tot}}(\rho_1^{*},\rho_2^{*})\right)}.$$

Recall the NE and global optimum point computed in (5) and (8), the PoA performance is provided in the following corollary:

Corollary 1:

$$\text{PoA} = \begin{cases} \frac{32}{27}, & \alpha<1; \\ \frac{2\alpha(\alpha+1)^{\frac{2}{\alpha}+2}(\alpha+2)^{\frac{1}{\alpha}+2}}{(2\alpha+1)^{\frac{2}{\alpha}+4}}, & \alpha\ge 1. \end{cases} \tag{11}$$

Remark 2: Corollary 1 suggests when $\alpha\to\infty$, PoA $= O(\alpha)\to\infty$, indicating throughput is much more important to user 2, he will try to occupy the bandwidth by making λ_2 closer to the service rate μ. In this case, the user 1 has little bandwidth and a high PAoI. As a result, PoA goes to infinity. To avoid such scenarios, in section IV, we propose an incentive packet-dropping scheme so that user 2 is pushed to control his sending rate, which leads to a larger social welfare for this system.

IV. Incentive Packet-Dropping Scheme

Since selfish behaviours are observed, we need incentives (e.g., pricing or packet drop for congestion control) to prevent any user from occupying the bandwidth without limits. A simple packet-dropping incentives mechanism in proposed in [9] so that the selfish users will control their sending rate. The Price of Anarchy for "power" maximization problem is shown to be reduced significantly. Inspired by the previous work, we aim at studying efficient packet-dropping mechanism to increase the value of the global utility function.

Let $P_d(\rho)$ be the packet-dropping probability, where $\rho=\rho_1+\rho_2$ is the total system utilization ratio of all the users and let $P_r(\rho)=1-P_d(\rho_1+\rho_2)$ remaining probability if packet-dropping incentives is implemented. The effective system utilization ratio of user i can be computed by $\rho_i^{(e)}=\rho_i P_r(\rho)$.

In the presence of packet-dropping incentives, for simplicity, recall that $\rho_{\text{tot}}=\rho_1+\rho_2$ is the sum of system utilization ratio, then the non-cooperative rate control game can be formulated as:

Problem 3 (Opt-Source 1 with packet-dropping mechanism):

$$\min_{\rho_1 P_r(\rho)\in[0,1-\rho_2 P_r(\rho))} U_1(\rho_1,\rho_2) = \frac{1}{\rho_1 P_r(\rho)} + \frac{1}{1-\rho P_r(\rho)}.$$

Problem 4 (Opt-Source 2 with packet-dropping mechanism):

$$\max_{\rho_2 P_r(\rho)\in[0,1-\rho_1 P_r(\rho))} U_2(\rho_1,\rho_2) = \begin{cases} (\rho_2 P_r(\rho))^\alpha(1-(\rho_1+\rho_2)P_r(\rho)), & \alpha<1; \\ (\rho_2 P_r(\rho))(1-(\rho_1+\rho_2)P_r(\rho))^{1/\alpha}, & \alpha\ge 1. \end{cases} \tag{12}$$

Denote the new uncooperative rate control game when the active packet drop rate is function $P_d(\cdot)=P(\cdot)$ as game G_p. The goal is to design an appropriate and simple packet-loss function $P(\cdot)$ so that the total utility function of the Nash Equilibrium point can be closer to the global optimum point of the original game $U_{\text{tot}}^{OPT}(\rho_1^{**},\rho_2^{**})$.

A. Linear Dropping Function

In this paper, we focus on designing a linear packet-dropping scheme to improve the total utility performance of the NE. Linear dropping scheme means when the total utilization ratio exceeds the threshold r, the remaining probability changes linearly with the slope $-A(A>0)$ as shown in Fig. 2.

Intuitively, for packet-dropping mechanism that help to reduce the PoA performance, the NE points of the new distributed rate control game with packet-dropping incentives, i.e., $(\rho_1^{d,*},\rho_2^{d,*})$, should be in the linear region of the dropping

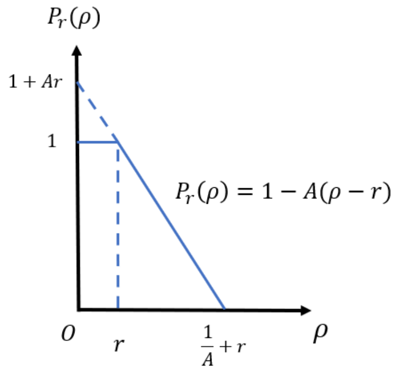

Fig. 2. A Linear Remaining Probability Function

function, i.e., $\rho_1^{d,*} + \rho_2^{d,*} \geq r$. Besides, for user 1, the NE point should satisfy $\frac{\partial U_1^d(\rho_1^{d,*},\rho_2^{d,*})}{\partial \rho_2} = 0$, i.e.,

$$\begin{aligned}&\frac{\partial U_1^d(\rho_1^{d,*},\rho_2^{d,*})}{\partial \rho_1}\\ =&-\frac{Pr(\rho_1^{d,*}+\rho_2^{d,*})+\rho_1^{d,*}Pr'(\rho_1^{d,*}+\rho_2^{d,*})}{(\rho_1^{d,*}P_r(\rho_1^{d,*}+\rho_2^{d,*}))^2}\\ &+\frac{P_r(\rho_1^{d,*}+\rho_2^{d,*})+(\rho_1^{d,*}+\rho_2^{d,*})Pr'(\rho_1^{d,*}+\rho_2^{d,*})}{(1-(\rho_1^{d,*}+\rho_2^{d,*})P_r(\rho_1^{d,*}+\rho_2^{d,*}))^2}\\ =&0. \end{aligned} \tag{13a}$$

For user 2, the NE point should satisfy $\frac{\partial \log U_2^d(\rho_1^{d,*},\rho_2^{d,*})}{\partial \rho_2} = 0$, i.e.,

$$\begin{aligned}&\frac{\partial \log U_2^d(\rho_1^{d,*},\rho_2^{d,*})}{\partial \rho_2}\\ \propto&\frac{\partial}{\partial \rho_2}\left(\alpha \log\left(\rho_2^{d,*}P_r(\rho_1^{d,*}+\rho_2^{d,*})\right)\right.\\ &\left.+\log\left(1-(\rho_1^{d,*}+\rho_2^{d,*})P_r(\rho_1^{d,*}+\rho_2^{d,*})\right)\right)\\ =&\alpha\frac{P_r(\rho_1^{d,*}+\rho_2^{d,*})+\rho_2^{d,*}P_r'(\rho_1^{d,*}+\rho_2^{d,*})}{\rho_2^{d,*}P_r(\rho_1^{d,*}+\rho_2^{d,*})}\\ &-\frac{P_r(\rho_1^{d,*}+\rho_2^{d,*})+(\rho_1^{d,*}+\rho_2^{d,*})P_r'(\rho_1^{d,*}+\rho_2^{d,*})}{1-(\rho_1^{d,*}+\rho_2^{d,*})P_r(\rho_1^{d,*}+\rho_2^{d,*})}\\ =&0. \end{aligned} \tag{13b}$$

To simplify further analysis, let $P_0 = P_r(\rho_1^{d,*} + \rho_2^{d,*})$ be the remaining probability at the NE points. By plugging $A = P_r'(\rho_1+\rho_2), \forall \rho_1+\rho_2 \in [r, r+\frac{1}{A}]$ into the above equations, we investigate the necessary condition of the "*effective*" system utilization ratio $\hat{\rho}_i^{d,*} = \rho_i^{d,*}P_0$:

Proposition 3: For a fixed linear packet-dropping function that starts to drop packets from r with descent rate A, if the corresponding NE exists $(\rho_1^{d,*}, \rho_2^{d,*})$ and satisfies on $\rho_1^{d,*} + \rho_2^{d,*} \in [r, r+\frac{1}{A}]$, then the effective system utilization ratio $(\hat{\rho}_1^{d,*}, \hat{\rho}_2^{d,*})$ corresponding to the NE points must satisfy the following equations:

$$\begin{aligned}&((\hat{\rho}_1^{d,*})^2-(1-\hat{\rho}_1^{d,*}-\hat{\rho}_2^{d,*})^2)\frac{{P_0}^2}{A}-\\ &\quad\hat{\rho}_1^{d,*}((\hat{\rho}_1^{d,*})^2+\hat{\rho}_1^{d,*}\hat{\rho}_2^{d,*}-(1-\hat{\rho}_1^{d,*}-\hat{\rho}_2^{d,*})^2)=0, \end{aligned} \tag{14a}$$

$$\begin{aligned}&(\alpha(1-\hat{\rho}_1^{d,*}-\hat{\rho}_2^{d,*})-\hat{\rho}_2^{d,*})\frac{{P_0}^2}{A}-\\ &\quad(\alpha\hat{\rho}_2^{d,*}(1-\hat{\rho}_1^{d,*}-\hat{\rho}_2^{d,*})-\hat{\rho}_2^{d,*}(\hat{\rho}_1^{d,*}+\hat{\rho}_2^{d,*}))=0. \end{aligned} \tag{14b}$$

Eq. (14a) provides the necessary condition for an NE if the slope of the linear dropping function is A. However, we will then prove that the global optimal point $(\rho_1^{**}, \rho_2^{**})$ is not a solution to Eq. (14a) for almost any A, and thus we have the following proposition:

Proposition 4: No linear dropping function can make PoA be 1 except for $\alpha = \frac{1+\sqrt{5}}{2}$.

Even though linear packet dropping schemes cannot incent the two uncooperative users to reach the global optimum for most of the values of α, we can design proper packet dropping incentive schemes so that the PoA performance is decreased. We then propose a linear packet-dropping scheme in Algorithm 1 and the performance is evaluated in simulations in subsection B:

Algorithm 1 Parameter Calculation for Incentives Packet-Dropping Scheme

Input: The parameter of the definition of "power", α
Output: The value of parameters A and r of the linear remaining probability function

Calculate the solution set $\mathcal{S}$ of Eq. (15)

$$\begin{aligned}&(2\alpha-\alpha\hat{\rho}_2^d)\hat{\rho}_1^{d2}+((\alpha+1)\hat{\rho}_2^{d2}-\alpha)\hat{\rho}_1^d\\ &+(\alpha+1)\hat{\rho}_2^{d3}-(2\alpha+1)\hat{\rho}_2^{d2}+\alpha\hat{\rho}_2^d=0. \end{aligned} \tag{15}$$

Initialize $(\hat{\rho}_1^{d,*}, \hat{\rho}_2^{d,*}) = (0,0), K^* = 0$.
for all point $(\hat{\rho}_1^{d,\mathcal{S}}, \hat{\rho}_2^{d,\mathcal{S}})$ in set $\mathcal{S}$ **do**
  Calculate the value of system total utilization of this point $U_{\text{tot}}(\hat{\rho}_1^{d,\mathcal{S}}, \hat{\rho}_2^{d,\mathcal{S}})$ and the parameter K as follows:

$$K=\frac{\hat{\rho}_1^{d,\mathcal{S}}(\hat{\rho}_1^{d,\mathcal{S}2}+\hat{\rho}_1^{d,\mathcal{S}}\hat{\rho}_2^{d,\mathcal{S}}-(1-\hat{\rho}_1^{d,\mathcal{S}}-\hat{\rho}_2^{d,\mathcal{S}})^2)}{\hat{\rho}_1^{d,\mathcal{S}2}-(1-\hat{\rho}_1^{d,\mathcal{S}}-\hat{\rho}_2^{d,\mathcal{S}})^2}. \tag{16}$$

  if $U_{\text{tot}}(\hat{\rho}_1^{d,\mathcal{S}}, \hat{\rho}_2^{d,\mathcal{S}}) > U_{\text{tot}}(\hat{\rho}_1^{d,*}, \hat{\rho}_2^{d,*})$ and $K > 0$ **then**
    Let $(\hat{\rho}_1^{d,*}, \hat{\rho}_2^{d,*}) = (\hat{\rho}_1^{d,\mathcal{S}}, \hat{\rho}_2^{d,\mathcal{S}}), K^* = K$.
  end if
end for
Calculate the total effective rate $\hat{\rho}_{\text{tot}}^{d,*} = \hat{\rho}_1^{d,*} + \hat{\rho}_2^{d,*}$.
Calculate the parameter A of the linear dropping function as:

$$A = \frac{1}{K^*}, \tag{17}$$

$$r = \hat{\rho}_{\text{tot}}^{d,*}. \tag{18}$$

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B. Simulations

We validate the performance of Algorithm 1 via numerical simulations. Fig. 3 depicts the performance of PoA graph in semilog coordinate system with the value of α changing. This linear dropping scheme makes the PoA decreased apparently. We find that this method makes sense when α is in a certain interval around 1. Besides, when $\alpha \to \infty$, PoA is finite with this scheme, i.e, the PoA is controlled under a certain bound by this linear dropping scheme, which is approximately 1.707. By numerical simulation we have proved the effectiveness of this linear dropping scheme.

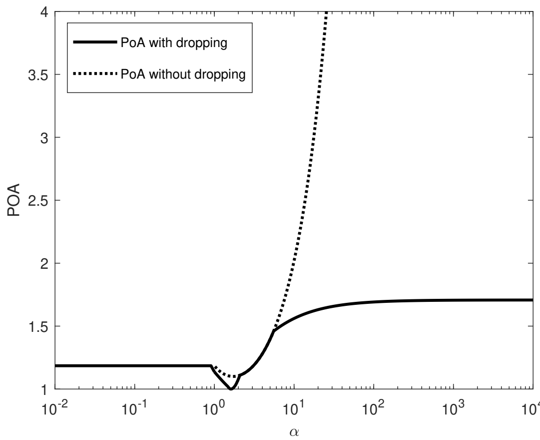

Fig. 3. PoA figure

V. Conclusion

In this paper, we consider the problem of rate controlling with heterogeneous QoS requirements. We propose an appropriate definition of the system total welfare in the case of two users with different QoS requirements. Since the original problem shows inefficiency by providing a large PoA in some case, we study the packet-dropping incentive mechanism. We design a linear dropping scheme, which is proved to be useful by numerical simulations. This method may be extended to the problem with more users.

Appendix A
Proof of Proposition 1

Proof: Notice that for fixed ρ_2, the second order derivative of average PAoI for user 1 satisfies:

$$\frac{\partial^2}{\partial \rho_1^2} C(\rho_1,\rho_2) = \frac{1}{\mu}\left(\frac{2}{\rho_1^3} + \frac{2}{(1-\rho_1-\rho_2)^3}\right) > 0, \quad \forall \rho_1 \in (0, 1-\rho_2). \tag{19}$$

Thus for fixed ρ_2, $C(\rho_1,\rho_2)$ is strictly convex for $\rho_1 \in (0, 1-\rho_2)$. As a result, the system utilization ratio ρ_1 that minimizes the average PAoI performance given ρ_2, denoted by $f_1(\rho_2)$ is unique. By forcing $\frac{\partial C(\rho_1,\rho_2)}{\partial \rho_1} = \frac{1}{\mu}\left(-\frac{1}{\rho_1^2} + \frac{1}{(1-\rho_1-\rho_2)^2}\right) = 0$, the optimum sampling/submitting utilization ratio for source 1 given ratio ρ_2 can be computed by:

$$f_1(\rho_2) = \frac{1-\rho_2}{2}. \tag{20}$$

Similar to [10], it can be proved that the optimum system utilization ratio source 2 should choose is $\frac{\alpha}{(1+\alpha)}\rho_1$. In order to keep the discussion self-contained, we derive the above statement as follows: Considering the utility function $U(\rho_1,\rho_2)$:

$$\begin{aligned}\frac{\partial U(\rho_1,\rho_2)}{\partial \rho_2} &= \mu^{1+\alpha}\left(\alpha\rho_2^{\alpha-1}(1-\rho_1-\rho_2) - \rho_2^{\alpha}\right) \\ &= \mu^{1+\alpha}\rho_2^{\alpha-1}\left(\alpha(1-\rho_1-\rho_2)-\rho_2\right).\end{aligned} \tag{21}$$

In interval $(0, 1-\rho_1)$, there exists only one solution $\rho_2 = \frac{\alpha}{(1+\alpha)}(1-\rho_1)$ so that $\frac{\partial U(\rho_1,\rho_2)}{\partial \rho_2} = 0$. Notice that the partial derivation $\frac{\partial U(\rho_1,\rho_2')}{\partial \rho_2} > 0, \forall \rho_2 \in (0, \frac{\alpha}{1+\alpha}(1-\rho_1))$ and $\frac{\partial U(\rho_1,\rho_2)}{\partial \rho_2} < 0, \forall \rho_2 \in (\frac{\alpha}{1+\alpha}(1-\rho_1), 1-\rho_1)$. Thus, for fixed ρ_1, the optimum system ratio source 2 should choose to maximize user 2's utility is:

$$f_2(\rho_1) = \frac{\alpha}{1+\alpha}(1-\rho_1). \tag{22}$$

The Nash equilibrium system utilization ratio pair (ρ_1^*, ρ_2^*) should satisfies:

$$f_1(\rho_2^*) = \rho_1^*, f_2(\rho_1^*) = \rho_2^*. \tag{23}$$

Plugging Eq. (20) and (22) into the above equation, we can then obtain the Nash equilibrium pair in Proposition 1. ■

Appendix B
Proof of Proposition 4

From Eq. (14a) and Eq. (14b) we can get an equation of ρ_1^e and ρ_2^e:

$$\frac{\hat\rho_1^{d,*2} - (1-\hat\rho_1^{d,*}-\hat\rho_2^{d,*})^2}{\alpha(1-\hat\rho_1^{d,*}-\hat\rho_2^{d,*}) - \hat\rho_2^{d,*}} = \frac{\hat\rho_1^{d,*}(\hat\rho_1^{d,*2} + \hat\rho_1^{d,*}\hat\rho_2^{d,*} - (1-\hat\rho_1^{d,*}-\hat\rho_2^{d,*})^2)}{\alpha\hat\rho_2^{d,*}(1-\hat\rho_1^{d,*}-\hat\rho_2^{d,*}) - \hat\rho_2^{d,*}(\hat\rho_1^{d,*}+\hat\rho_2^{d,*})}. \tag{24}$$

When $\alpha < 1$, let $(\hat\rho_1^{d,*}, \hat\rho_2^{d,*}) = (\frac{2}{3(\alpha+2)}, \frac{\alpha}{\alpha+2})$, the left hand side (LHS) of the above function can be simplified to:

$$\text{LHS} = \frac{\frac{4}{9(\alpha+2)^2} - \frac{16}{9(\alpha+2)^2}}{\frac{4\alpha}{3(\alpha+2)} - \frac{3\alpha}{3(\alpha+2)}} = -\frac{4}{\alpha(\alpha+2)}, \tag{25}$$

and the RHS becomes

$$\text{RHS} = \frac{\frac{2}{3(\alpha+2)}\left(\frac{4}{9(\alpha+2)^2} + \frac{6\alpha}{9(\alpha+2)^2} - \frac{16}{9(\alpha+2)^2}\right)}{\alpha\frac{\alpha}{\alpha+2}\frac{4}{\alpha+2} - \frac{\alpha}{\alpha+2}\frac{3\alpha+2}{3(\alpha+2)}} = \frac{4}{3\alpha(\alpha+2)}. \tag{26}$$

Apparently the simplified LHS≠RHS, so the global optimum point can not be a solution of the NE equations with linear dropping scheme when $\alpha < 1$.

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When $\alpha \geq 1$, let $(\hat{\rho}_1^{d,*}, \hat{\rho}_2^{d,*}) = (\frac{\alpha^2+\alpha}{(2\alpha+1)^2}, \frac{\alpha}{2\alpha+1})$,we have:

$$\text{LHS} = \frac{\frac{\alpha^2(\alpha+1)^2}{(2\alpha+1)^4} - \frac{(\alpha+1)^4}{(2\alpha+1)^4}}{\frac{\alpha(\alpha+1)^2}{(2\alpha+1)^2} - \frac{\alpha}{2\alpha+1}} = -\frac{(\alpha+1)^2}{\alpha^3(2\alpha+1)}, \quad (27)$$

$$\begin{aligned}\text{RHS} &= \frac{\frac{\alpha(\alpha+1)}{(2\alpha+1)^2}(\frac{\alpha^2(\alpha+1)^2}{(2\alpha+1)^4} + \frac{\alpha^2(\alpha+1)}{(2\alpha+1)^3} - \frac{(\alpha+1)^4}{(2\alpha+1)^4})}{\frac{\alpha^2}{2\alpha+1}\frac{(\alpha+1)^2}{(2\alpha+1)^2} - \frac{\alpha}{2\alpha+1}\frac{\alpha(3\alpha+2)}{(2\alpha+1)^2}} \\ &= \frac{(\alpha+1)^2(2\alpha^3 - \alpha^2 - 3\alpha - 1)}{\alpha(2\alpha+1)^3(\alpha^2 - \alpha - 1)}. \quad (28)\end{aligned}$$

Only $\alpha = \frac{1+\sqrt{5}}{2} \approx 1.618$ makes LHS = RHS, so the global optimum point can not be a solution of the NE equations with linear dropping scheme except for $\alpha = \frac{1+\sqrt{5}}{2}$.

Appendix C
Proof of Algorithm 1

Let $K = \frac{{P_0}^2}{A}$ in Eq. (14a):

$$\begin{aligned}&(\hat{\rho}_1^{d,*2} - (1 - \hat{\rho}_1^{d,*} - \hat{\rho}_2^{d,*})^2)K - \\ &\hat{\rho}_1^{d,*}(\hat{\rho}_1^{d,*2} + \hat{\rho}_1^{d,*}\hat{\rho}_2^{d,*} - (1 - \hat{\rho}_1^{d,*} - \hat{\rho}_2^{d,*})^2) = 0 \\ \Rightarrow K &= \frac{\hat{\rho}_1^{d,*}(\hat{\rho}_1^{d,*2} + \hat{\rho}_1^{d,*}\hat{\rho}_2^{d,*} - (1 - \hat{\rho}_1^{d,*} - \hat{\rho}_2^{d,*})^2)}{\hat{\rho}_1^{d,*2} - (1 - \hat{\rho}_1^{d,*} - \hat{\rho}_2^{d,*})^2}, \quad (29)\end{aligned}$$

and this is Eq. (16).

In Eq. (14b) there is:

$$\begin{aligned}&(\alpha(1 - \hat{\rho}_1^{d,*} - \hat{\rho}_2^{d,*}) - \hat{\rho}_2^{d,*})\frac{{P_0}^2}{A} - \\ &(\alpha\hat{\rho}_2^{d,*}(1 - \hat{\rho}_1^{d,*} - \hat{\rho}_2^{d,*}) - \hat{\rho}_2^{d,*}(\hat{\rho}_1^{d,*} + \hat{\rho}_2^{d,*})) = 0 \\ \Rightarrow K &= \frac{\alpha\hat{\rho}_2^{d,*}(1 - \hat{\rho}_1^{d,*} - \hat{\rho}_2^{d,*}) - \hat{\rho}_2^{d,*}(\hat{\rho}_1^{d,*} + \hat{\rho}_2^{d,*})}{\alpha(1 - \hat{\rho}_1^{d,*} - \hat{\rho}_2^{d,*}) - \hat{\rho}_2^{d,*}}. \quad (30)\end{aligned}$$

From (29) and (30) we get the equation of $\hat{\rho}_1^{d,*}$ and $\hat{\rho}_2^{d,*}$ which is irrelevant to the parameter K:

$$\begin{aligned}&\frac{\hat{\rho}_1^{d,*}(\hat{\rho}_1^{d,*2} + \hat{\rho}_1^{d,*}\hat{\rho}_2^{d,*} - (1 - \hat{\rho}_1^{d,*} - \hat{\rho}_2^{d,*})^2)}{\hat{\rho}_1^{d,*2} - (1 - \hat{\rho}_1^{d,*} - \hat{\rho}_2^{d,*})^2} \\ &= \frac{\alpha\hat{\rho}_2^{d,*}(1 - \hat{\rho}_1^{d,*} - \hat{\rho}_2^{d,*}) - \hat{\rho}_2^{d,*}(\hat{\rho}_1^{d,*} + \hat{\rho}_2^{d,*})}{\alpha(1 - \hat{\rho}_1^{d,*} - \hat{\rho}_2^{d,*}) - \hat{\rho}_2^{d,*}}. \quad (31)\end{aligned}$$

Simplify this equation, and finally we get:

$$\begin{aligned}&(2\alpha - \alpha\hat{\rho}_2^{d,*})\hat{\rho}_1^{d,*2} + ((\alpha+1)\hat{\rho}_2^{d,*2} - \alpha)\hat{\rho}_1^{d,*} \\ &+ (\alpha+1)\hat{\rho}_2^{d,*3} - (2\alpha+1)\hat{\rho}_2^{d,*2} + \alpha\hat{\rho}_2^{d,*} = 0, \quad (32)\end{aligned}$$

and this is Eq. (15).

According to Algorithm 1, $(\hat{\rho}_1^{d,*}, \hat{\rho}_2^{d,*})$ is a solution of the equation above with the smallest PoA. Now we need to determine A and K as we have gotten the value of K^* and $\hat{\rho}_{\text{tot}}^{d,*} = \hat{\rho}_1^{d,*} + \hat{\rho}_2^{d,*}$.

According to the definition, when the sending rate(not the "effective" one) is $\frac{\hat{\rho}_{\text{tot}}^{d,*}}{P_0}$, the remaining probability should be P_0, i.e,

$$P_0 = 1 - A(\frac{\hat{\rho}_{\text{tot}}^{d,*}}{P_0} - r). \quad (33)$$

Notice that $K^* = \frac{{P_0}^2}{A}$, so

$$\begin{aligned}P_0 &= 1 + \frac{{P_0}^2}{K^*}r - \frac{{P_0}^2}{K^*}\frac{\hat{\rho}_{\text{tot}}^{d,*}}{P_0} \\ &= 1 + \frac{{P_0}^2}{K^*}r - \frac{\hat{\rho}_{\text{tot}}^{d,*}}{K^*}P_0. \quad (34)\end{aligned}$$

We want fewer packets to be dropped, so we try setting the remaining probability in the NE point as 1. Let $P_0 = 1$, then

$$\begin{aligned}1 &= 1 + \frac{r}{K^*} - \frac{\hat{\rho}_{\text{tot}}^{d,*}}{K^*} \\ \Rightarrow r &= \hat{\rho}_{\text{tot}}^{d,*}, \quad (35)\end{aligned}$$

and

$$A = \frac{1}{K^*}. \quad (36)$$

References

[1] S. Kaul, R. Yates, and M. Gruteser, "Real-time status: How often should one update?" in *2012 Proceedings IEEE INFOCOM*, March 2012, pp. 2731–2735.

[2] Y. Sun, E. Uysal-Biyikoglu, R. D. Yates, C. E. Koksal, and N. B. Shroff, "Update or wait: How to keep your data fresh," *IEEE Transactions on Information Theory*, vol. 63, no. 11, pp. 7492–7508, Nov 2017.

[3] I. Kadota, A. Sinha, E. Uysal-Biyikoglu, R. Singh, and E. Modiano, "Scheduling policies for minimizing age of information in broadcast wireless networks," *IEEE/ACM Transactions on Networking*, vol. 26, no. 6, pp. 2637–2650, Dec 2018.

[4] H. Tang, J. Wang, L. Song, and J. Song, "Scheduling to minimize age of information in multi-state time-varying networks with power constraints," in *2019 57th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*.

[5] J. Sun, Z. Jiang, S. Zhou, and Z. Niu, "Age-optimal scheduling for heterogeneous traffic with timely-throughput constraint," in *IEEE INFOCOM 2020 - IEEE Conference on Computer Communications Workshops (INFOCOM WKSHPS)*, 2020, pp. 317–322.

[6] A. Garnaev, J. Zhong, W. Zhang, R. D. Yates, and W. Trappe, "Proportional fair information freshness under jamming," in *Wired/Wireless Internet Communications - 17th IFIP WG 6.2 International Conference, WWIC 2019, Bologna, Italy, June 17-18, 2019, Proceedings*, ser. Lecture Notes in Computer Science, M. D. Felice, E. Natalizio, R. Bruno, and A. Kassler, Eds., vol. 11618. Springer, 2019, pp. 91–102. [Online]. Available: https://doi.org/10.1007/978-3-030-30523-9_8

[7] L. Huang and E. Modiano, "Optimizing age-of-information in a multi-class queueing system," in *2015 IEEE International Symposium on Information Theory (ISIT)*, June 2015, pp. 1681–1685.

[8] "Gateway Congestion Control Survey," RFC 1254, Aug. 1991. [Online]. Available: https://rfc-editor.org/rfc/rfc1254.txt

[9] Y. Gai, H. Liu, and B. Krishnamachari, "A packet dropping-based incentive mechanism for m/m/1 queues with selfish users," in *2011 Proceedings IEEE INFOCOM*, 2011, pp. 2687–2695.

[10] C. Douligeris and R. Mazumdar, "A game theoretic perspective to flow control in telecommunication networks," *Journal of the Franklin Institute*, vol. 329, no. 2, pp. 383 – 402, 1992. [Online]. Available: http://www.sciencedirect.com/science/article/pii/001600329290041E

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