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Optimizing Age of Information in Multicast Unilateral Networks

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***Abstract*—In this work, we consider a scenario where a time sensitive source is broadcasted to multiple receivers by a base station (BS) over unilateral networks. A recently proposed metric–the *Age of Information* (AoI) is adopted to measure data freshness from the perspective of receiver. Unlike previous work, we consider that the BS receives no feedback from receivers and thus broadcasts every fixed interval. We derived the optimum fixed interval such that the average AoI can be minimized. Our work suggests that, when the transmission delay is highly random, the optimum fixed interval is larger than the expected transmission delay so that success delivery to more users can be guaranteed. Both theoretical analysis and simulation results show that without feedback, the average AoI performance following the proposed policy is near to transmission policy that utilizes receiver feedback.**

***Index Terms*—Age of Information, Multicast Network, Unilateral Network**

I. Introduction

Data freshness plays an important role in autonomous vehicles and the Internet of Things (IoT) networks. In such scenarios, the base station (BS) or the central controller broadcasts time-sensitive information to multiple receivers via multi-cast network. Due to the large number of access nodes and limited communication resources, feedback from all the receivers is impossible. This work accounts for this scenario by designing multi-casting strategy in the absence of receiver feedback.

The metric, *Age of Information* (AoI), namely the time elapsed since the newest sample at the receiver is generated, has been proposed and widely adopted to measure data freshness from the perspective of the receiver [1]. In a multi-user setup, scheduling to minimize average AoI has been studied recently in [2]–[10]. When the transmission experiences delay or packet drop, Lyapunov optimization and Whittle's index approach have been used to design uni-cast transmission strategy [2]–[6]. In multi-cast networks with random transmission delay, it is found that starting a new transmission after feedback from a group of receivers is beneficial to minimize the average AoI performance [7]–[11]. However, the above works assume feedback from the receiver to be instantaneous and perfect. Recent work [12] studies AoI minimization strategy when feedback from the receiver may be incorrect. It is revealed that feedback erasure increases the average AoI and thus studying transmission strategy under imperfect feedback is a challenging task.

To address the challenge, we consider a BS multi-casts time-sensitive information to receivers through channels with random delay. The goal is to provide insight into the transmission strategy design in the absence of user feedback. The model can be used to model a variety of real-time applications like web caching and autonomous vehicles. The main contribution is that we optimize the average AoI by proposing a strategy that multi-casts every fixed interval. We derived the optimum multi-casting interval and show in simulation that by following our proposed policy, no feedback causes subtle AoI growth compared with updating strategies based on receiver feedback in [10].

The remainder of the paper is organized as follows. The system model and optimization problem is stated in Section II. Section III proposes the fixed interval multi-casting strategy and derives the optimum interval to achieve the minimum average AoI. Numerical simulations are provided in Section IV and Section V draws the conclusion.

II. Problem Formulation

A. Network Model

We model the system as a BS broadcasting update packets of a time-sensitive source to N receivers, as depicted in Fig. 1. We consider a continuous time scenario. If the BS decides to broadcast a packet at time t_j, then it sends a snapshot of the source at time t_j, and successful transmission to receiver i experiences a random delay $X_{i,j}$. In this work, we assume $X_{i,j}$ is an independent random variable that follows a shifted exponential distribution, whose probability density function $f(x)$ is as follows:

$$f(x)=\begin{cases}\lambda e^{-\lambda(x-c)}, & x\geq c;\\ 0, & \text{else},\end{cases} \tag{1}$$

where c represents the fixed or the minimum transmission delay determined by transmission distance, power, etc. Due to limited channel capacity, we assume the link from the BS

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to the receiver can transmit only one packet at a time. Thus, if the BS decides to broadcast a new packet, the previous transmission from BS to receiver i will be suspended if it is still not received and the transmission of the new update packet begins immediately. Different from previous work [9], [10], [13], we consider the broadcasting network to be unilateral and the receivers are not allowed to send any feedback to the source.

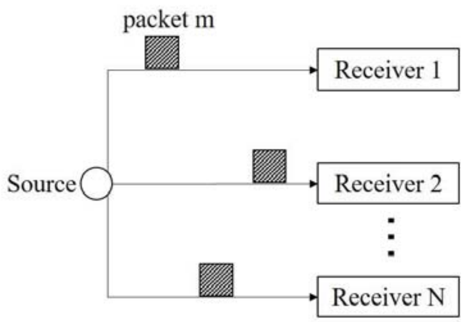

Fig. 1: System model

B. AoI Metric

Notice that due to transmission delay, each receiver cannot have the newest information about the time-sensitive source. To measure how "*fresh*" the data is from the perspective of each receiver, the metric, *Age of Information* (AoI) [1] is proposed. By definition, the AoI measures the difference between the current time t and the time-stamp when the freshest information at the receiver is generated. Let $u_i(t)$ be the generation time-stamp of the freshest information received by receiver i. Then

$$u_i(t) = \arg\max\{t_j | t_j + X_{i,j} \leq t\}.$$

Thus the AoI of receiver i at time t, denoted by $\Delta_i(t)$, can be computed as follows:

$$\Delta_i(t) = t - u_i(t). \tag{2}$$

To better illustrate the concept of AoI, a sample path of AoI evolution is plotted in Fig. 2.

C. Optimization Problem

Our goal is to design a transmission strategy in the absence of receiver feedback so that each receiver can possess fresh information about the source. The data freshness of receivers in the network is measured by the average AoI, which can be computed as follows since each receiver is identical:

$$\overline{\Delta} = \lim_{T\to\infty} \frac{1}{T} \int_0^T \Delta(t)\, dt. \tag{3}$$

We aim at minimizing $\overline{\Delta}$ by designing broadcasting strategy, i.e., the time to broadcast update $\{t_1, t_2, \cdots\}$.

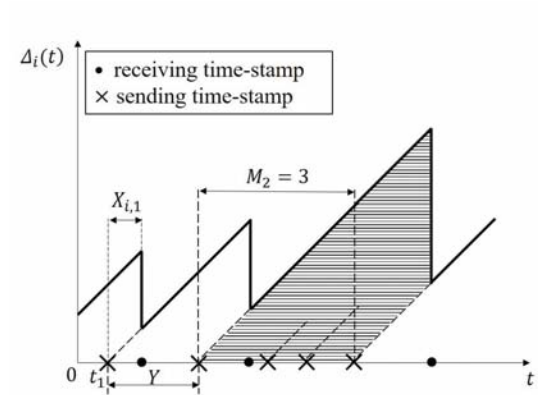

Fig. 2: A sample path of AoI evolution. The transmission starting time-stamps and receiving time-stamps are marked in solid circles and crosses, respectively.

III. Problem Resolution

A. Average AoI under Fixed Interval Policy

In this work, we consider broadcasting with fixed intervals, i.e., there exists a constant Y such that $t_{j+1} - t_j = Y, \forall j \geq 1$.

Theorem 1: Under fixed interval policy with Y, the average AoI under shifted exponential delay can be computed by:

$$\overline{\Delta} = \frac{1}{\lambda} + \frac{Y}{2} + \frac{c}{1 - e^{-\lambda(Y-c)}}. \tag{4}$$

Proof: The average AoI can be computed by summing up the isosceles trapezoids under the AoI evolution curve depicted in Fig. 2. Each trapezoid can be divided into a parallelogram and an isosceles right triangle. Let M_k be the number of broadcasting intervals between the generation time-stamp of the k-th received packet and the $(k+1)$-th received packet. Then the bottom edge of the parallelogram and the right-angle side of the triangle equal to M_kY. And the height of the parallelogram equals the transmission delay of the k-th received packet $X_{i,k}$. Then the size of the trapezoid S_k can be computed by:

$$S_k = M_kY \cdot X_{i,k} + \frac{1}{2}M_k^2Y^2. \tag{5}$$

Let K_T be the number of received packets by the receiver at time T, then the average AoI can be written out as follows:

$$\begin{aligned}\overline{\Delta} &= \lim_{T\to\infty} \frac{1}{T} \int_{t=0}^{T} \Delta(t)dt \\ &= \lim_{T\to\infty} \frac{\sum_{k=1}^{K_T} S_k}{\sum_{k=1}^{K_T} M_kY} \\ &\overset{(a)}{=} \frac{\mathbb{E}[M_kX_{i,k}|X_{i,k} \leq Y]}{\mathbb{E}[M_k]} + \frac{E[M_k^2]Y}{2\mathbb{E}[M_k]} \\ &\overset{(b)}{=} \mathbb{E}[X_{i,k}|X_{i,k} \leq Y] + \frac{E[M_k^2]Y}{2\mathbb{E}[M_k]},\end{aligned} \tag{6}$$

where (a) is obtained by plugging Eq. (5) into Eq. (3) and (b) is obtained because M_k is independent of transmission delay $X_{i,k}$.

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According to the distribution of Eq. (1), the expected transmission delay when $X_{i,k} \le Y$ can be computed by:

$$\begin{aligned}&\mathbb{E}[X_{i,k}|X_{i,k} \le Y]\\&= \frac{\int_0^Y xf(x)\,dx}{\int_0^Y f(x)\,dx}\\&= \frac{\lambda c - \lambda Y e^{-\lambda(Y-c)} + 1 - e^{-\lambda(Y-c)}}{\lambda(1-e^{-\lambda(Y-c)})}. \end{aligned} \tag{7}$$

Let $p = \int_0^Y f(x)dx = 1 - e^{-\lambda(Y-c)}$ be the probability that the transmission delay is smaller than Y and hence the packet has been received successfully. The number of updates between intervals M_k follows an independent geometric distribution parameterized by p. Thus, we can compute $\frac{\mathbb{E}[M_k^2]}{2\mathbb{E}[M_k]}$ as follows:

$$\frac{\mathbb{E}[M_k^2]}{\mathbb{E}[M_k]} = \frac{2-p}{p} = \frac{1+e^{-\lambda(Y-c)}}{1-e^{-\lambda(Y-c)}}. \tag{8}$$

Finally, plugging Eq. (7) and Eq. (8) into Eq. (6), we can compute the average AoI as follows:

$$\begin{aligned}\overline{\Delta} &= \frac{1}{\lambda} + \frac{c}{1-e^{-\lambda(Y-c)}} - \frac{Ye^{-\lambda(Y-c)}}{1-e^{-\lambda(Y-c)}}\\&+ \frac{Y(1+e^{-\lambda(Y-c)})}{2(1-e^{-\lambda(Y-c)})}\\&= \frac{1}{\lambda} + \frac{c}{1-e^{-\lambda(Y-c)}} + \frac{Y}{2}.\end{aligned}$$

And that is the end of all the proof. ■

B. Determination of the Optimum Interval Y

Next, we aim at minimizing $\overline{\Delta}$ by choosing Y. First, we show that for given λ and c, the average AoI in Eq. (4) is a convex function for $Y > c$. The convexity is obtained by investigating its second order derivative:

$$\begin{aligned}\frac{\mathrm{d}^2\overline{\Delta}}{\mathrm{d}Y^2} &= \frac{\mathrm{d}}{\mathrm{d}Y}\left(\frac{\mathrm{d}\overline{\Delta}}{\mathrm{d}Y}\right)\\&= \frac{\mathrm{d}}{\mathrm{d}Y}\left(\frac{1}{2} - \frac{\lambda c e^{-\lambda(Y-c)}}{(1-e^{-\lambda(Y-c)})^2}\right)\\&= \frac{\lambda^2 c e^{-\lambda(Y-c)}\left[1+e^{-\lambda(Y-c)}\right]}{(1-e^{-\lambda(Y-c)})^3}.\end{aligned} \tag{9}$$

When the fixed interval $Y > c$, the denominator in Eq. (9) $(1-e^{-\lambda(Y-c)})^3 > 0$, indicating the convexity of $\overline{\Delta}$. According to Eq. (1), if the chosen fixed interval Y is smaller than the smallest transmission delay c, none of the receivers can receive the update packet successfully, which leads to infinite average AoI and is thus far from optimum. Hence, the optimum interval $Y^* > c$. Due to the convexity at (c, ∞), the sufficient and necessary condition for Y^* is:

$$\left.\frac{\mathrm{d}\overline{\Delta}}{\mathrm{d}Y}\right|_{Y=Y^*} = \frac{1}{2} - \frac{\lambda c e^{-\lambda(Y^*-c)}}{(1-e^{-\lambda(Y^*-c)})^2} = 0. \tag{10}$$

Solving Eq. (10) yields the optimum interval, i.e.,

$$Y^* = c - \frac{\ln(\lambda c + 1 - \sqrt{\lambda^2 c^2 + 2\lambda c})}{\lambda}. \tag{11}$$

Plugging Eq. (11) into Eq. (4), the minimum average AoI $\overline{\Delta}$ can be computed by:

$$\begin{aligned}\overline{\Delta}^* &= \frac{1}{\lambda} + \frac{c}{2} - \frac{1}{2\lambda}\ln(\lambda c + 1 - \sqrt{\lambda^2c^2 + 2\lambda c})\\&+ \frac{c}{\sqrt{\lambda^2c^2+2\lambda c} - \lambda c}.\end{aligned} \tag{12}$$

Remark: A transmission policy that utilizes transmission feedback from the receivers called the *Earliest k Stopping policy* is proposed in [10]. That policy optimizes the average AoI by computing the optimum number of ACKs k, which depends on channel parameters c and λ, such that a new transmission starts once k out of n receivers have received the current packet. The average AoI Δ_k^* obtained by the *Earliest k Stopping policy* is the same as Eq. (12) after approximation.

IV. Simulation Results

In this section, we provide simulation results to illustrate the performance of the proposed algorithm.

First, we compare the average AoI by applying the proposed fixed interval policy, denoted by π_Y^* and the Earliest k Stopping policy, denoted by π_{Ek} in [10] in different scenarios. In Fig. 3, we consider two scenarios with $n = 3$ and $n = 20$, respectively. The performance of fixed interval policy π_Y^* and the Earliest k Stopping policy π_{Ek} are marked in red dashed lines and blue solid lines, respectively. From Fig. 3, the average AoI difference between the proposed fixed interval policy π_Y^* and the Earliest k Stopping policy π_{Ek} decreases when the number of receivers n increases. This phenomenon suggests a good average AoI performance by choosing the optimum fixed interval Y when the number of receivers in the network is large.

To figure out how fixed interval policy achieves such good AoI performance without feedback from the receivers, we plot the optimum interval as a function of c and λ in Fig. 4. For further analysis, Fig. 4 also depicts the expected delay of a single receiver and the expected transmission delay of the first k receivers with 20 receivers in total. The optimum interval is marked in red dashed lines, and the expected delay of a single receiver and first k receivers are marked in green dotted lines and blue solid lines respectively. Fig. 4 reveals the optimum interval is near to the expected delay of the first k success transmissions. Moreover, as n becomes larger, the variance of the update interval also decreases. Thus both two factors lead to compatible performance between the proposed fixed interval policy and the Earliest k stopping policy.

Notice that the random delay consists of two parts, the fixed part c and the stochastic part whose expected value is $\frac{1}{\lambda}$. The optimum interval increases monotonically when the fixed part c increases and decreases with λ. Moreover, the product of channel parameters λc describes the relative sizes of fixed and stochastic delay. From Fig. 4, when $\lambda c \ll 1$, i.e., the fixed delay is much smaller than the stochastic one (see the left bottom of Fig. 4), the optimum interval is smaller than the expectation $\mathbb{E}[X_{i,j}]$. This phenomenon suggests leveraging fixed delay can make up for the potential large stochastic delay

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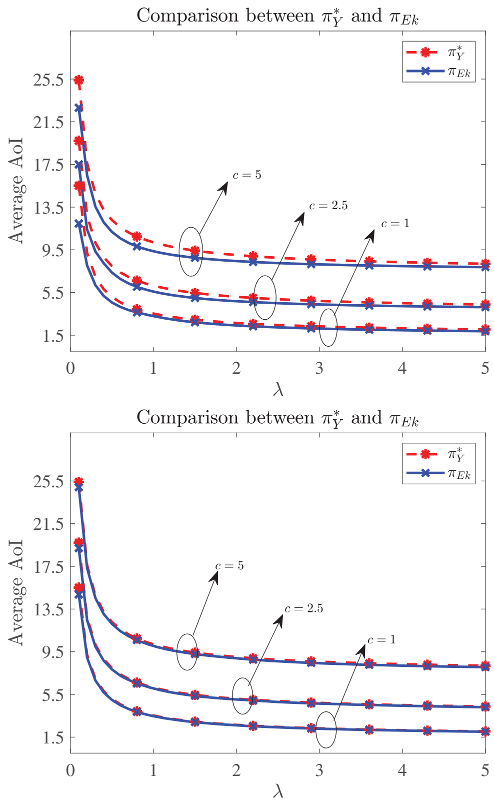

Fig. 3: Comparison of the proposed fixed interval policy π_Y^* and the Earliest k Stopping policy π_{Ek} in a multi-cast network with $n = 3$ (top) and $n = 20$ (bottom).

through frequent updating; when $\lambda c \gg 1$ (e.g., the right top of Fig. 4), the optimum interval is relatively equal to and larger than $\mathbb{E}[X_{i,j}]$. Then, to minimize the average AoI of all the receivers when the fixed delay is much larger than the stochastic one, the BS tends to wait for a much longer interval than the expected delay so that the success delivery of more receivers can be guaranteed.

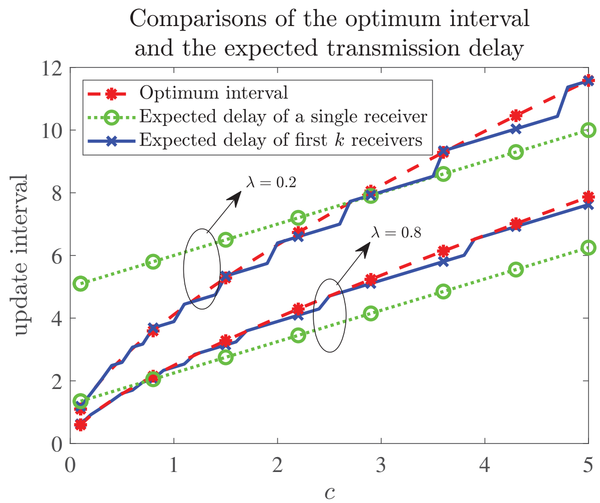

Fig. 4: Comparisons of the optimum interval and the expected transmission delay, where we consider the expected delay of a single receiver and the first k success transmissions out of $n = 20$ receivers.

V. Conclusion

In this work, we study the fixed interval transmission strategy that minimizes the average AoI in a unilateral multi-cast network. We derived the optimum interval and computed the average AoI under such strategy. Both theoretical analysis and simulation results showed that, in the absence of feedback from the receivers, the optimum fixed interval transmission strategy achieves a compatible average AoI performance as the *Earliest k Stopping policy*. Interesting extensions to the work includes studying transmission strategies when the delay distribution of receivers are not identical.

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