Sampling of the Wiener Process for Remote Estimation over a Channel with Unknown Delay Statistics

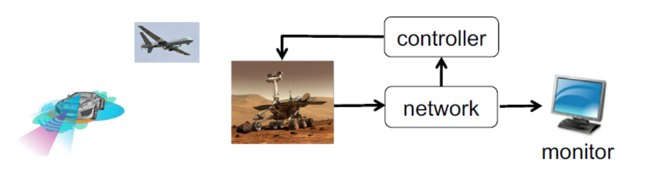
Haoyue Tang[1], Yin Sun[2] and Leandros Tassiulas[1]

[1]Yale Institute of Network Science, Yale University
[2]Department of ECE, Auburn University

Contents

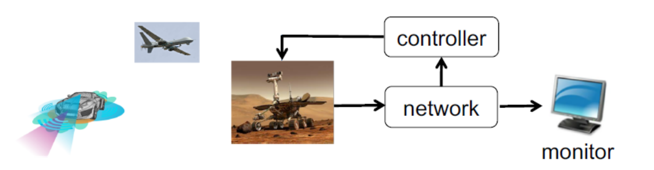
Background

Real-time service requires fresh data



Background

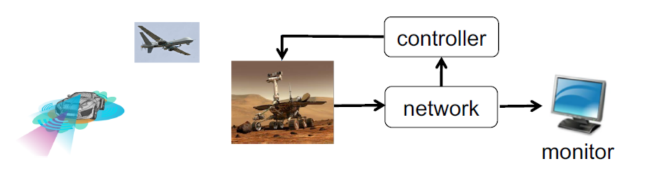
Real-time service requires fresh data



Real-time data analytics becomes important

Background

Real-time service requires fresh data

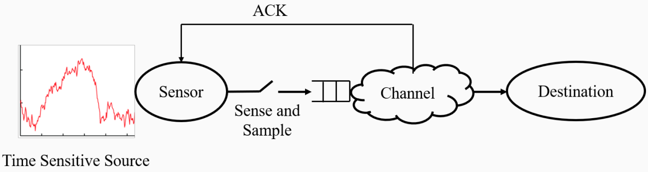


Real-time data analytics becomes important

Research Problem

How to optimize information freshness of a time-varying process?

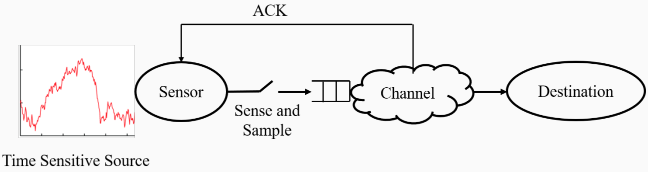
This Work: A Special Example–the Wiener Process



- Point-to-point link: **Sensor** senses a Wiener process and submits samples to the **Destination**
- **Channel**: FIFO queue with i.i.d. transmission times
- **Feedback**: zero-delay ACK
  - Busy/Idle state of the channel is known to the sensor.

The transmission delay D_k of sample k in the channel is i.i.d following distribution $\mathbb{P}_D$.

This Work: A Special Example–the Wiener Process



- Point-to-point link: **Sensor** senses a Wiener process and submits samples to the **Destination**
- **Channel**: FIFO queue with i.i.d. transmission times
- **Feedback**: zero-delay ACK
  - Busy/Idle state of the channel is known to the sensor.

The transmission delay D_k of sample k in the channel is i.i.d following distribution $\mathbb{P}_D$.

Challenge

We consider the delay distribution $\mathbb{P}_D$ is unknown before making sampling decisions.

Existing Work and Main Contributions

Existing work

- Online Age of Information (AoI) Minimization
  - min AoI using RL/Bandits (Atay et al. (2021); Leng and Yener (2021), ...) Theoretic analysis are missing.
  - min AoI under unknown delay statistics (Tang et al. (2022)) Requires prior knowledge about mean and moment of $\mathbb{P}_D$
- Off-line MSE minimum sampling Sun et al. (2020)

Existing Work and Main Contributions

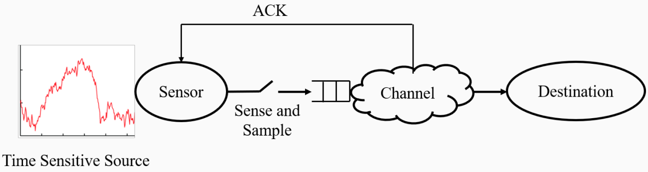
Existing work

- Online Age of Information (AoI) Minimization
  - min AoI using RL/Bandits (Atay et al. (2021); Leng and Yener (2021), ...) Theoretic analysis are missing.
  - min AoI under unknown delay statistics (Tang et al. (2022)) Requires prior knowledge about mean and moment of $\mathbb{P}_D$
- Off-line MSE minimum sampling Sun et al. (2020)

Contributions

- Propose an online learning algorithm to minimize the MSE based on Robbins-Monro.
- Derive the convergence rate of the proposed online algorithm. No prior information on $\mathbb{P}_D$ is required.
- Establish converse result for any causal sampling algorithm. Verify minimax order optimal using non-parametric statistics.

Problem Formulation: Minimizing MSE



MMSE Estimator: $\hat{X}_t = \underbrace{X_{i(t)}, i(t) := \arg\min_i \{R_i \leq t\}}_{\text{value of the most recently received sample}}$.

Optimization Problem

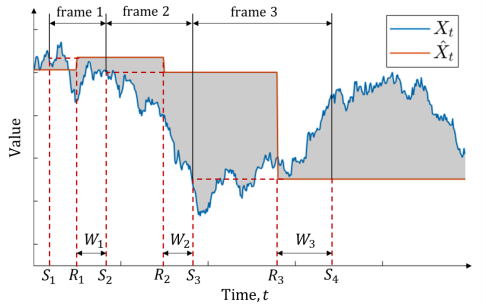
$$\mathsf{mse}_{\mathsf{opt}} \triangleq \inf_{\pi \in \Pi} \limsup_{T \to \infty} \frac{1}{T} \mathbb{E}\left[\int_{t=0}^{T} (X_t - \hat{X}_t)^2 \mathrm{d}t\right], \tag{1a}$$

$$\text{s.t.} \quad \liminf_{K \to \infty} \frac{1}{K} \mathbb{E}\left[\sum_{k=1}^{K} \underbrace{(S_{k+1} - S_k)}_{k\text{-th sampling interval}}\right] \geq \frac{1}{f_{\max}}. \tag{1b}$$

Optimum policy $\pi^\star$ achieving $\mathsf{mse}_{\mathsf{opt}}$ **:Sun et al. (2020)**

After receiving ACK of sample k, wait $W_k \geq 0$ to take sample $(k+1)$.

Structure of $\pi^\star$:



Frame k

interval between k-th and the $(k+1)$-th sample.

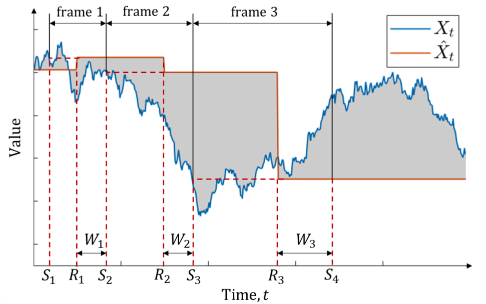
Optimal Policy: a threshold structure

$$W_k = \inf\{t \geq 0| \underbrace{|X_{R_k+t} - X_{S_k}|}_{\text{Signal Difference w.r.t start of frame } k} \geq \sqrt{3(\gamma^\star + \nu^\star)}\}.$$

Signal Difference

w.r.t start of frame k

Structure of $\pi^\star$:



Frame k

interval between k-th and the $(k+1)$-th sample.

Optimal Policy: a threshold structure

$$W_k = \inf\{t \geq 0 | \underbrace{|X_{R_k+t} - X_{S_k}|}_{\text{Signal Difference w.r.t start of frame } k} \geq \sqrt{3(\gamma^\star + \nu^\star)}\}.$$

where,

$$\gamma^\star = \limsup_{K\to\infty} \frac{\mathbb{E}_{\pi^\star}\left[\sum_{k=1}^{K} \frac{1}{6}(X_{S_{k+1}} - X_{S_k})^4\right]}{\mathbb{E}_{\pi^\star}\left[\sum_{k=1}^{K}(S_{k+1} - S_k)\right]} = \frac{\text{avg. reward}}{\text{avg. frame length}}$$

$\nu^\star$: satisfy sampling frequency cons.

Online Algorithm: Finding $\gamma^\star$ and $\nu^\star$ (1)

$$\gamma^\star = \limsup_{K\to\infty} \mathbb{E}_{\pi^\star}\left[\sum_{k=1}^{K} \underbrace{\frac{1}{6}(X_{S_{k+1}} - X_{S_k})^4}_{\text{Reward } E_k \text{ in frame } k}\right] \Big/ \mathbb{E}_{\pi^\star}\left[\sum_{k=1}^{K} \underbrace{(S_{k+1} - S_k)}_{\text{Length of frame } k}\right]$$

Problem: Guessing the root of $\overline{E}(\gamma^\star) - \gamma^\star \overline{L}(\gamma^\star) = 0$

Solution: Stochastic Approximation using Robbins-Monro

In frame k:

- **Sampling**: assume current γ_k and ν_k is correct, i.e., wait

$$W_k = \inf\{t \geq 0 | |X_{R_k+t} - X_{S_k}| \geq \sqrt{3(\gamma_k + \nu_k)}\}$$

- Compute reward E_k and length L_k:

$$E_k = (X_{S_{k+1}} - X_{S_k})^4/6, L_k = D_k + W_k.$$

- **Update γ_k:** $\gamma_{k+1} = (\gamma_k + \eta_k(E_k - \gamma_k L_k))^+$.

Online Algorithm: Finding $\gamma^\star$ and $\nu^\star$ (2)

$\nu^\star$: Virtual queue recording sampling interval violation

Update dual optimizer ν_k:

$$\nu_{k+1} = \left(\nu_k + \frac{1}{V}(L_k - \frac{1}{f_{\max}})\right)^+. \quad (2)$$

Analysis (1)–Approximating $\gamma^\star$:

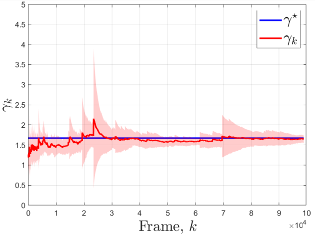
Updating policy represented by $\{\gamma_k\}$

If the delay D is fourth order bounded,

$$\lim_{K\to\infty} \gamma_k = \gamma^\star, \text{w.p.1}, \tag{3}$$

and the approximation error:

$$\mathbb{E}[(\gamma_k - \gamma^\star)^2] = \mathcal{O}(1/k). \tag{4}$$



Implication: The optimum policy $\pi^\star$ is learned almost surely.

Proof Challenge: Sequence γ_k is in open set $[0, \infty)$, i.e.

$$\gamma_{k+1} = (\gamma_k + \eta_k E_k - \eta_k \gamma_k L_k))^+ .$$

Queueing system is also unbounded, i.e.,

$$\text{Queue}[t+1] = [\text{Queue}[t] + \text{Arrival}[t] - \text{Departure}[t]]^+$$

Drift analysis from heavy traffic can overcome the challenge!

Analysis (2)–MSE Performance

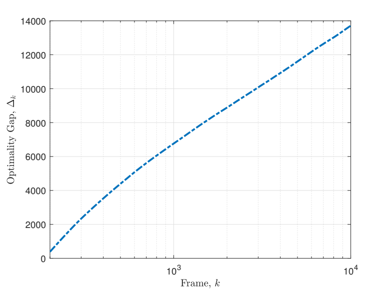
Learning Rate

If the delay D is forth order bounded, the time-average MSE of the proposed algorithm converges to $\mathrm{mse_{opt}}$ almost surely, i.e.,

$$\lim_{K\to\infty} \frac{\int_{t=0}^{S_{K+1}} (X_t - \hat{X}_t)^2 \mathrm{d}t}{S_{K+1}} = \overline{\mathcal{E}}_{\pi^\star}, \text{w.p.1}, \tag{5}$$

and the convergence rate:

$$\Delta_K := \mathbb{E}\left[\int_{t=0}^{S_{K+1}} (X_t - \hat{X}_t)^2 \mathrm{d}t\right] - \mathbb{E}[S_{K+1}]\overline{\mathcal{E}}_{\pi^\star} = \mathcal{O}\left(\ln K\right). \tag{6}$$



Convergence Rate Analysis:
the perturbed ODE method

Analysis (3)–Converse Result

Converse Result

The average AoI regret for any causal policy π satisfies:

$$\inf_{\pi} \sup_{\mathbb{P}} \left(\mathbb{E}\left[\int_{t=0}^{S_{K+1}} (X_t - \hat{X}_t)^2 \mathrm{d}t \right] - \mathbb{E}[S_{K+1}] \overline{\mathcal{E}}_{\pi^\star} \right) = \Omega\left(\ln K\right). \quad (7)$$

Step 1: Link $\Delta_k = \mathbb{E}\left[\int_{t=0}^{S_{K+1}} (X_t - \hat{X}_t)^2 \mathrm{d}t\right] - \mathbb{E}[S_{K+1}]\overline{\mathcal{E}}_{\pi^\star}$ with error $(\hat{\gamma}_k - \gamma^\star)^2$.

Step 2: Bounding $\mathbb{E}[(\hat{\gamma}_k - \gamma^\star)^2]$ using Le Cam's two-point method.

Why a minimax bound?

We do not restrict $\mathbb{P}_D$ belong to any specific family (e.g., non-exponential).

Simulations (1)

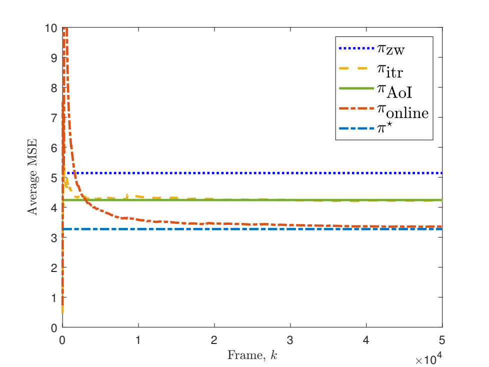


Figure 1: MSE evolution with frame $\mathbb{E}[\int_{t=0}^{S_{K+1}}(X_t - \hat{X}_t)^2 \mathrm{d}t]/\mathbb{E}[S_{K+1}]$ (left)

Signal-aware optimum sampling is much better than signal-ignorant AoI optimal sampling.

Simulations (2)

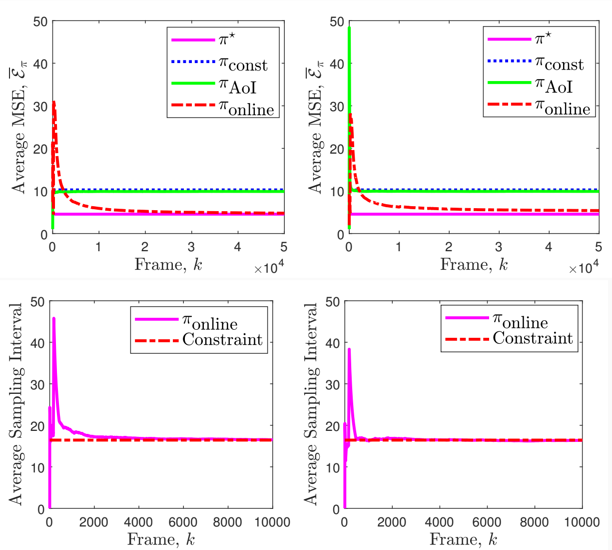


Figure 2: The time average MSE (up) and sampling interval (down).

π_{online} can satisfy the sampling frequency constraint, smaller V converges faster.

Conclusions

- Contribution:
  - The first to use Robbins-Monro to AoI related problems Neely (2021).
  - Develop a new method for proving convergence rate of stochastic approximation algorithm in an **open** set.
  - Converse bound for online algorithm using non-parametric statistics.
- See our paper!
  H. Tang, Y. Sun and L. Tassiulas, "Sampling of the Wiener Process for Remote Estimation over a Channel with Unknown Delay Statistics", Mobihoc2022

Thank you! Questions?

References

E. U. Atay, I. Kadota, and E. H. Modiano. Aging wireless bandits: Regret analysis and order-optimal learning algorithm. In *WiOpt*, pages 57–64. IFIP, 2021.

S. Leng and A. Yener. An actor-critic reinforcement learning approach to minimum age of information scheduling in energy harvesting networks. In *ICASSP 2021 - 2021 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pages 8128–8132, 2021. doi: 10.1109/ICASSP39728.2021.9415110.

M. J. Neely. Fast learning for renewal optimization in online task scheduling. *Journal of Machine Learning Research*, 22(279):1–44, 2021.

Y. Sun, Y. Polyanskiy, and E. Uysal. Sampling of the wiener process for remote estimation over a channel with random delay. *IEEE Transactions on Information Theory*, 66(2):1118–1135, 2020.

H. Tang, Y. Chen, J. Sun, J. Wang, and J. Song. Sending timely status updates through channel with random delay via online learning. In *IEEE INFOCOM 2022 - IEEE Conference on Computer Communications*, pages 1819–1827, 2022. doi: 10.1109/INFOCOM48880.2022.9796970.