Scheduling to Minimize Age of Synchronization in Wireless Broadcast Networks with Random Updates

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ISIT, July 9, 2019

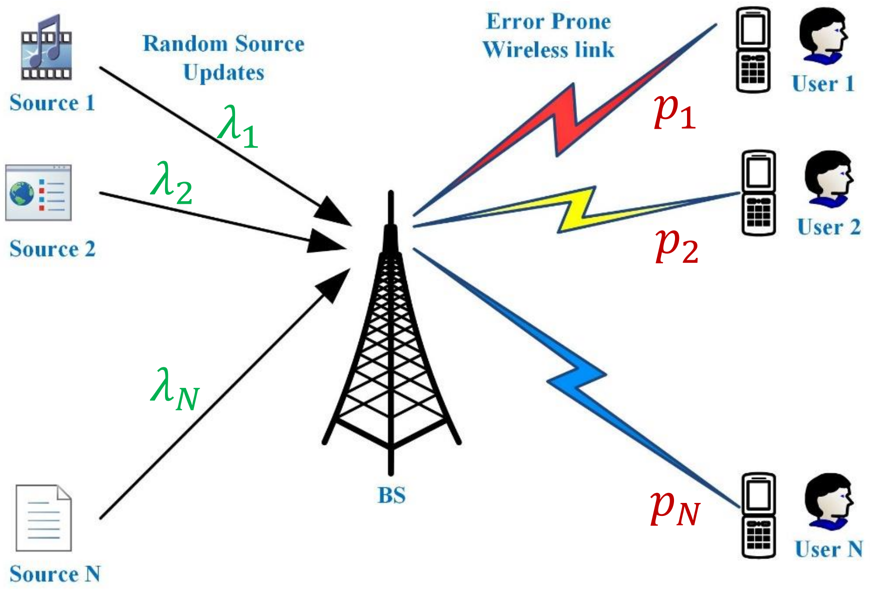
Outline

- Problem Formulation
  - Network model
  - Metric introduction and comparison

- **Scheduling Policies**
  - **Markov Decision Process**
  - **Whittle’s Index**

- Numerical Simulations

Network Model



- BS broadcasts random information updates to users
- In each slot t, scheduling to user n succeeds with probability p_n
- An update of source n arrives with probability λ_n
- BS can only keep one snap shot of each source

What is the data freshness metric if there is no information change between two packet update?

- We measure the data freshness of user n with AoS $s_n(t)$ at the beginning of each slot

Metric Introduction--Definition

Age of Synchronization

- The time elapsed since the freshest message became desynchronized

Age of Information

- The time elapsed since the generation time-stamp of the freshest message

Suppose g_i, r_i are the generation and receiving time-stamp of the i-th update packet

The index of the freshest information at time t is:

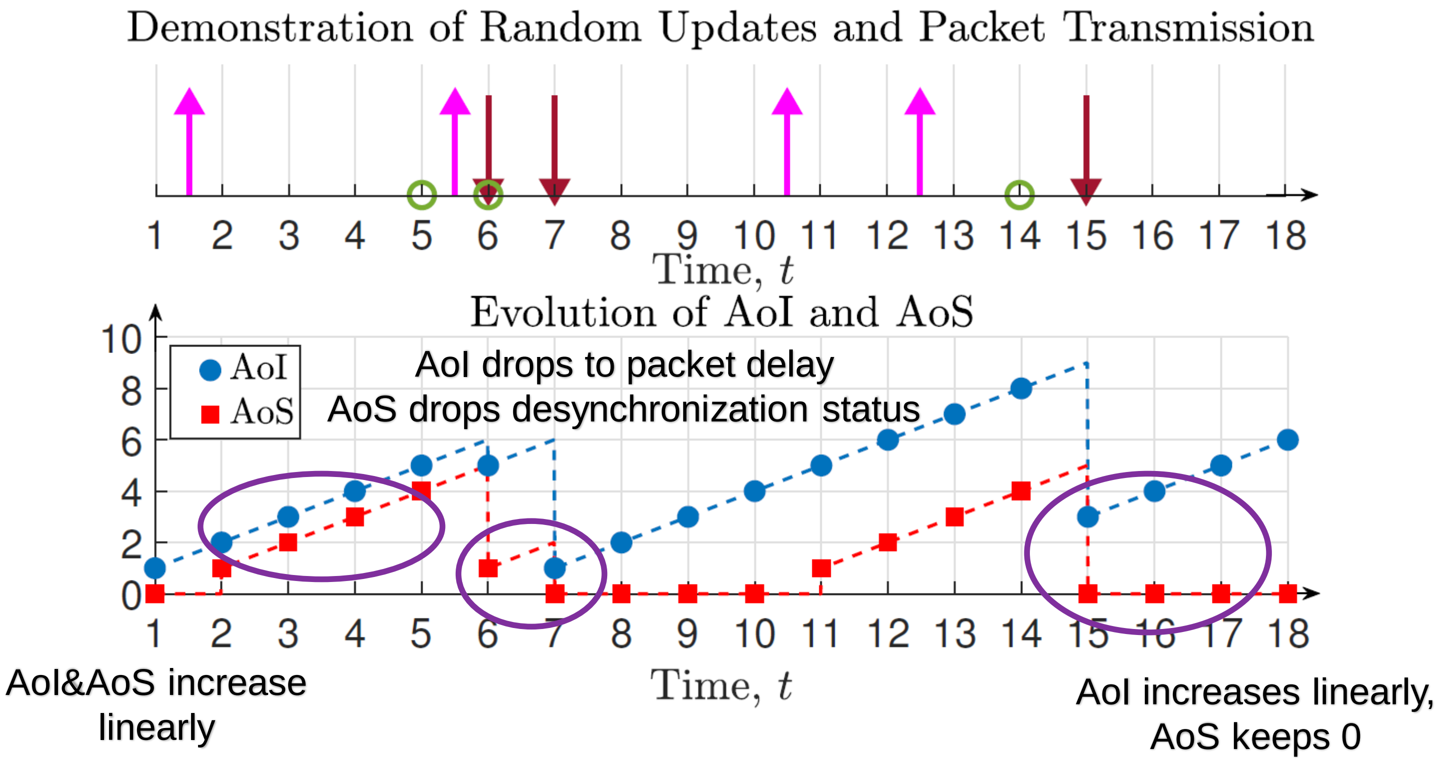
$$q(t) = \arg\max_{n \in \mathbb{N}^+} \{r_n | r_n \leq t\}$$

Then:

$$\mathrm{AoS}(t) = \left(t - g_{q(t)+1}\right)^+, \mathrm{AoI}(t) = t - g_{q(t)}$$

J. Zhong, R. D. Yates, and E. Soljanin, “Two Freshness Metrics for Local Cache Refresh,” in 2018 IEEE International Symposium on Information Theory (ISIT), Jun. 2018, pp. 1924–1928.

Metric Introduction--Comparisons



Differences: AoS– use source as a reference

AoI– Inter-update generation duration is taken into account

Network Model—Age of Synchronization

- At the beginning of each slot, BS select user n, broadcasts the freshest information of source n $[u_n(t) = 1]$
- If transmission succeeds, packet will be received at the end of slot
- If the message at user n is desynchronized $s_n(t) \neq 0$:
  - User n is not scheduled $u_n(t) = 0$, $s_n(t+1) = s_n(t) + 1$
  - User n is scheduled $u_n(t) = 1$
    - Transmission fails w.p. $1 - p_n$: $s_n(t+1) = s_n(t) + 1$
    - Transmission succeeds w.p. p_n, $s_n(t+1)$ will also be determined by new packet arrival, thus:

$$s_n(t+1) = 1, \mathrm{w.p.}\, \lambda_n p_n; s_n(t+1) = 0, \mathrm{w.p.}\, (1-\lambda_n)p_n$$

- If the message at user n is synchronized $s_n(t) = 0$:

$$s_n(t+1) = 1, \mathrm{w.p.}\, \lambda_n; s_n(t+1) = 0, \mathrm{w.p.}\, 1-\lambda_n$$

When $\lambda_n = 1$, AoS=AoI

Network Model—Problem Formulation

- Goal: design a non-anticipated scheduling policy subject to interference constraint to minimize average AoS at the beginning of each slot

$$\pi^* = \arg \min_{\pi \in \Pi_{NA}} \lim_{T \to \infty} \frac{1}{NT} \mathbb{E}_\pi \left[\sum_{t=1}^{T} \sum_{n=1}^{N} s_n(t) \right],$$

---Objective Function

$$\text{s.t.} \sum_{n=1}^{N} u_n(t) \leq 1.$$

---Interference Constraint

Scheduling Policies—MDP(1)

- State: the current AoS of each node
  - $\mathbf{s}(t) = [s_1(t), \cdots, s_N(t)]$ (countable but infinite)
- Action: $\mathbf{u}(t) = [u_1(t), \cdots, u_N(t)]$
- Transition probability: $\Pr(\mathbf{s}'|\mathbf{s}, \mathbf{u}) = \prod_{n=1}^{N} \Pr(s_n'|s_n, u_n)$
- One-step cost: the increment of the average AoS:

$$C(\mathbf{s}(t), \mathbf{u}(t)) = \frac{1}{N} \sum_{n=1}^{N} s_n(t)$$

- The goal of the MDP is to minimize the average cost over infinite horizon

Scheduling Policies—MDP(2)

- Infinite state space?—A truncated MDP
  - Setting an upper bound on AoS, $x_n(t) = \max\{s_n(t), S_{\max}\}$
  - Refine the probability transfer function, cost function
- The optimum policy to the truncated MDP can be obtained by policy iteration, let $\pi(\mathbf{x})$ be the obtained optimum policy
- In each slot t, observe $s_n(t)$ of each user and compute $x_n(t)$, scheduling decision is made by: $\mathbf{u}(t) = \pi(\mathbf{x})$
- **Problem: High computational complexity!**

Scheduling Policies—Whittle's Index(1)

- To adopt the Restless Multi-arm Bandit Framework, we decouple each user and add a scheduling penalty C
- Then we consider the decoupled sub-problem:

$$\min_{\pi \in \Pi_{NA}} \lim_{T \to \infty} \frac{1}{T} \mathbb{E}_{\pi} \left[\sum_{t=1}^{T} s(t) + Cu(t) \right]$$

- Properties:
  - The optimum solution holds a threshold structure, i.e., if it's optimum to schedule at s, then for all state $s' > s$ the optimum strategy is to schedule the user
  - The threshold is a non-decreasing function of C

$$\tau = \left\lfloor \left(\frac{5}{2} - \frac{1}{p} - \frac{1}{\lambda} \right) + \sqrt{\left(\frac{5}{2} - \frac{1}{p} - \frac{1}{\lambda} \right)^2 + 2\left(\frac{C}{p} + \frac{1-\lambda}{\lambda} \frac{1-p}{p} \right) + 2\frac{1-p}{p}} \right\rfloor$$

Indexability is guaranteed

Scheduling Policies—Whittle's Index(2)

- Deviation of Whittle's Index:

$$W(s) = \frac{\left(F_{s+1}(0) - F_s(0)\right)}{\left(\xi_1^{(s)} - \xi_1^{(s+1)}\right)/p}$$

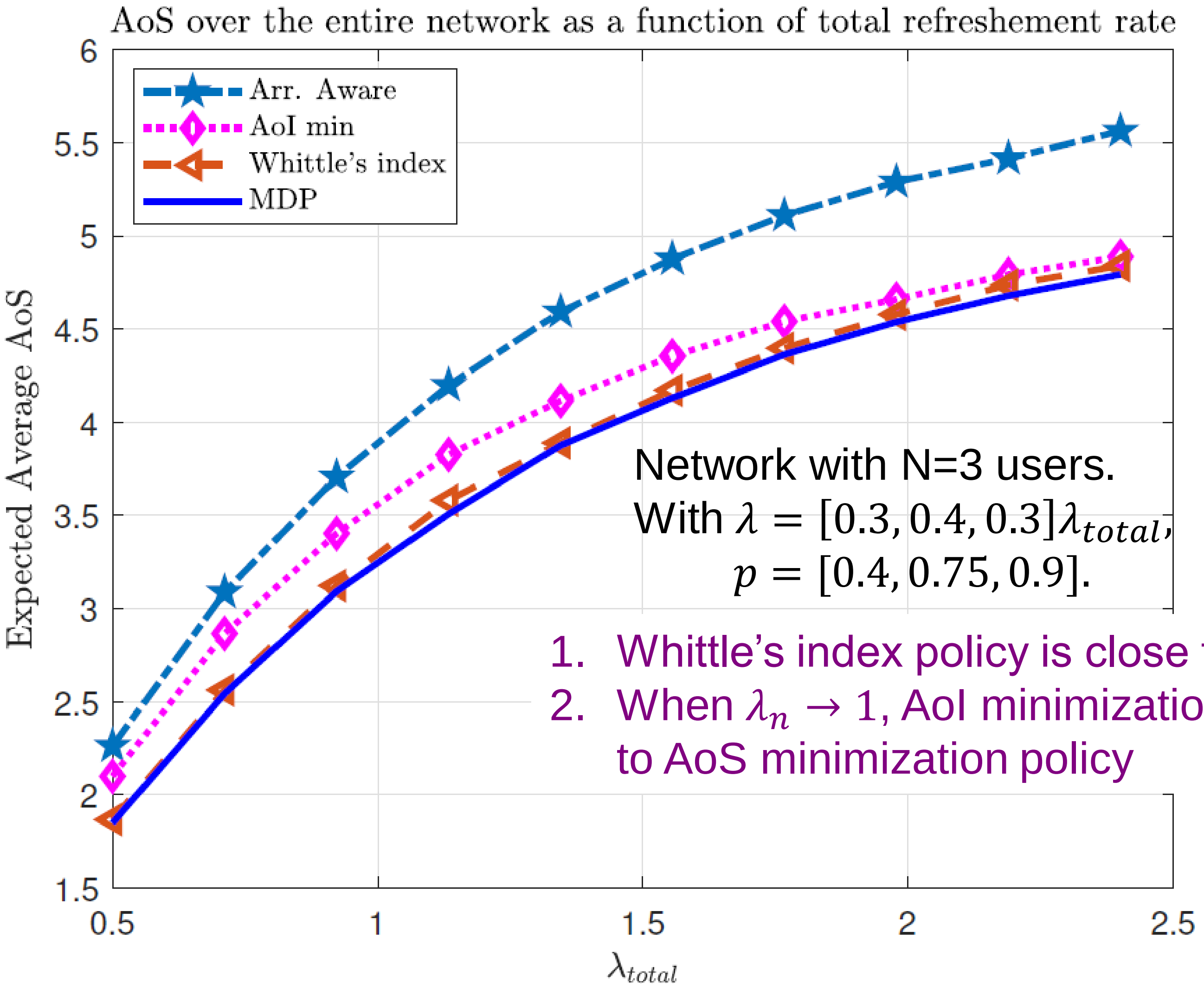
where $F_s(C) = \frac{s(s-1)}{2}\xi_1^{(s)} + \frac{\xi_1^{(s)}}{p}\left(\frac{1}{p} - 1\right) + \frac{\xi_1^{(s)}}{p}(s + C)$ is the total cost if apply s as threshold,

and $\xi_1^{(s)} = 1/(\frac{1-\lambda}{\lambda} + s + \frac{1}{p} - 1)$ is the probability that the bandit staying in state 1 if apply threshold policy τ

- Index Policy: in each time slot, select the node with the largest index $W_n\left(s_n(t)\right)$

J. Gittins, K. Glazebrook, and R. Weber, Multi-armed bandit allocation indices. John Wiley & Sons, 2011.

Simulations

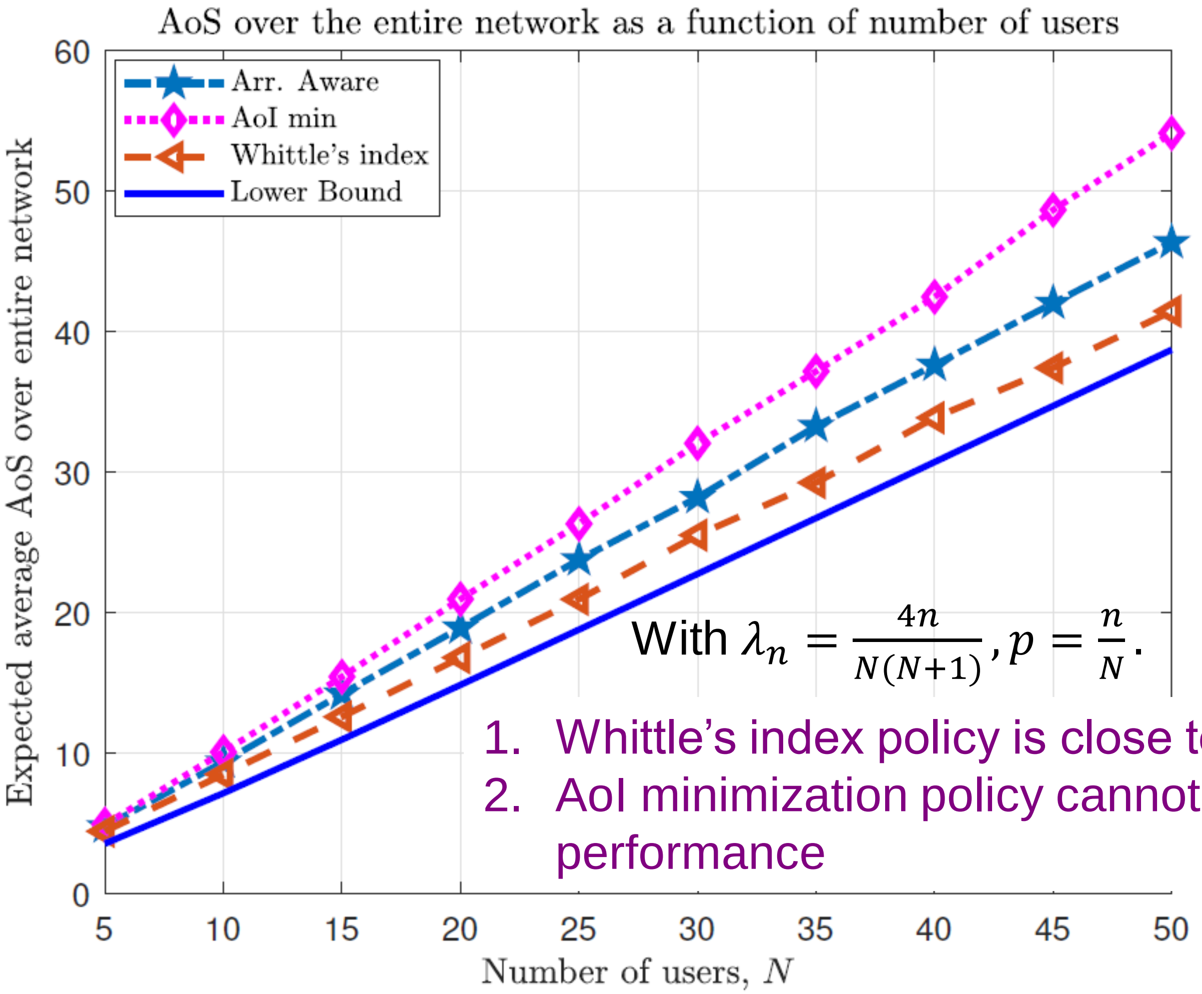


Network with N=3 users.
With $\lambda = [0.3, 0.4, 0.3]\lambda_{total}$,
$p = [0.4, 0.75, 0.9]$.

1. Whittle's index policy is close to truncated MDP
2. When $\lambda_n \to 1$, AoI minimization policy is similar to AoS minimization policy

J. Sun, Z. Jiang, S. Zhou, and Z. Niu, "Optimizing information freshness in broadcast network with unreliable links and random arrivals: An approximate index policy," in to appear IEEE INFOCOM 2019 - IEEE Conference on Computer Communications Workshops (INFOCOM WKSHPS), April 2019.

Simulations



With $\lambda_n = \frac{4n}{N(N+1)}, p = \frac{n}{N}$.

1. Whittle's index policy is close to the lower bound
2. AoI minimization policy cannot guarantee a good AoS performance

J. Sun, Z. Jiang, S. Zhou, and Z. Niu, "Optimizing information freshness in broadcast network with unreliable links and random arrivals: An approximate index policy," in to appear IEEE INFOCOM 2019 - IEEE Conference on Computer Communications Workshops (INFOCOM WKSHPS), April 2019.

Thank you! Q&A

More details and proofs see our supplementary materials:
https://www.dropbox.com/s/ch6qhq1nhzroyey/draft.pdf?dl=0