Data Freshness Oriented Sampling Under Unknown Delay Statistics

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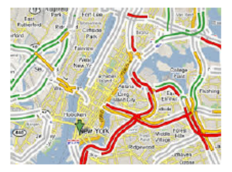
Introduction

Background: Emerging time-sensitive applications requires fresh data sampled from time-varying process.


Crowdsourcing

Online Learning (ads bidding)


Social networks

Challenges:

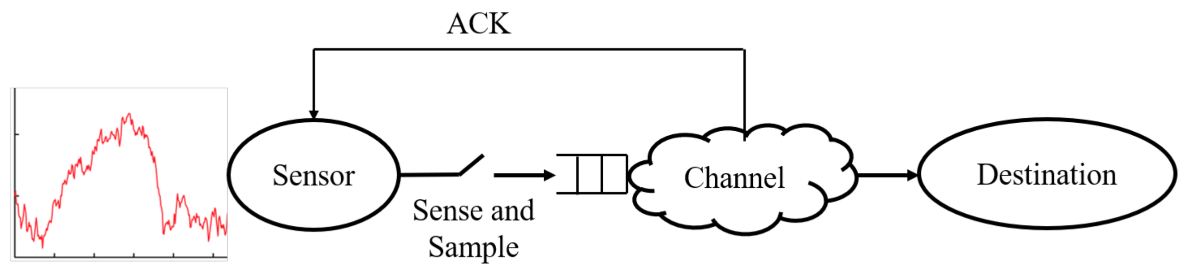
- Data freshness requirement requires system re-design.
- AoI, MSE can be unbounded. Theoretic analysis for data freshness oriented adaptive algorithm is missing.

Our work: **Fundamental limits and efficient algorithms** for sampling a Wiener process under **unknown** delay statistics.

- The first online learning algorithm without knowing the delay upper bound.
- New convergence result for stochastic approximation in an open space (the perturbed ODE and drift method).
- New converse result (Le Cam's Two point method).

System Model

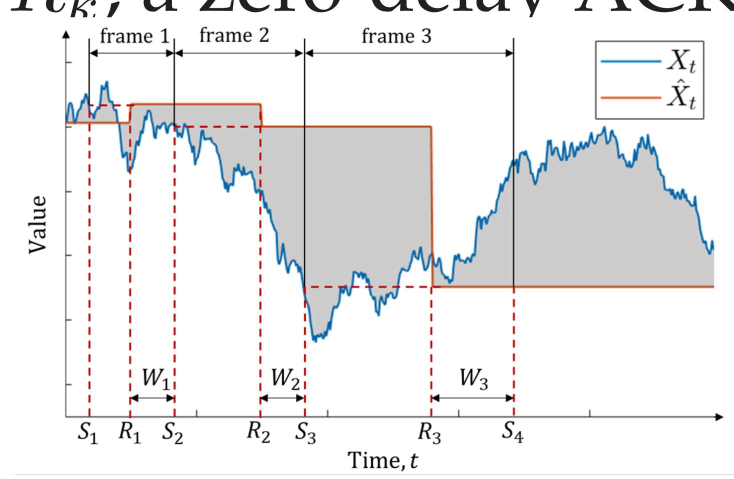
Sensor senses and submits the k-th sample to the **Destination** at time S_k:



- **Channel**: FIFO queue. The delay D_k of the k-th sample passing through the channel $D_k \overset{\text{i.i.d}}{\sim} \mathbb{P}_D$.
- **Feedback**: when sample k received by time R_k, a zero-delay ACK

Source: X_t is a Wiener process.

MMSE Estimator: $\hat{X}_t = X_{i(t)}$, $i(t) := \min\{j | R_j \leq t\}$: the index of the freshest sample at receiver.



$$\mathsf{mse}_{\text{opt}} \triangleq \inf_{\pi\in\Pi} \limsup_{T\to\infty} \mathbb{E}\left[\frac{1}{T}\int_{t=0}^{T}\left(\hat{X}_t - X_t\right)^2 \mathrm{d}t\right]$$

Proposed Algorithm π_{online}

Observation: Samples waiting in the queue are no longer fresh.

Solutions: After receive the ACK of sample k, wait for $W_k \geq 0$ to take sample $(k+1)$.

Optimum Offline Policy: $S_{k+1} = \inf\{t \geq S_k + D_k | |X_t - X_{S_k}| \geq \sqrt{3\gamma^\star}\}$, where $\gamma^\star = \mathsf{mse}_{\text{opt}} - \overline{D} = \frac{\text{avg. reward } R_k}{\text{avg. framelength} L_k}$.

Design Intuition: Finding root of equation $\overline{R}(\gamma^\star) - \gamma^\star \overline{L}(\gamma^\star)$.

- **Frame** k: Take sample $k+1$ at time $S_{k+1} = \inf_t\{|X_{S_k+t} - X_{S_k}| \geq \sqrt{3\gamma_k}\}$.
- **Approximate** γ_k **via Robbins-Monro:** $\gamma_{k+1} = \left(\gamma_k + \eta_k\left(R_k^4 - \gamma L_k\right)\right)^+$, where $\delta_k = X_{S_{k+1}} - X_{S_k}$, $R_k = \frac{1}{6}\delta X_k^4$.

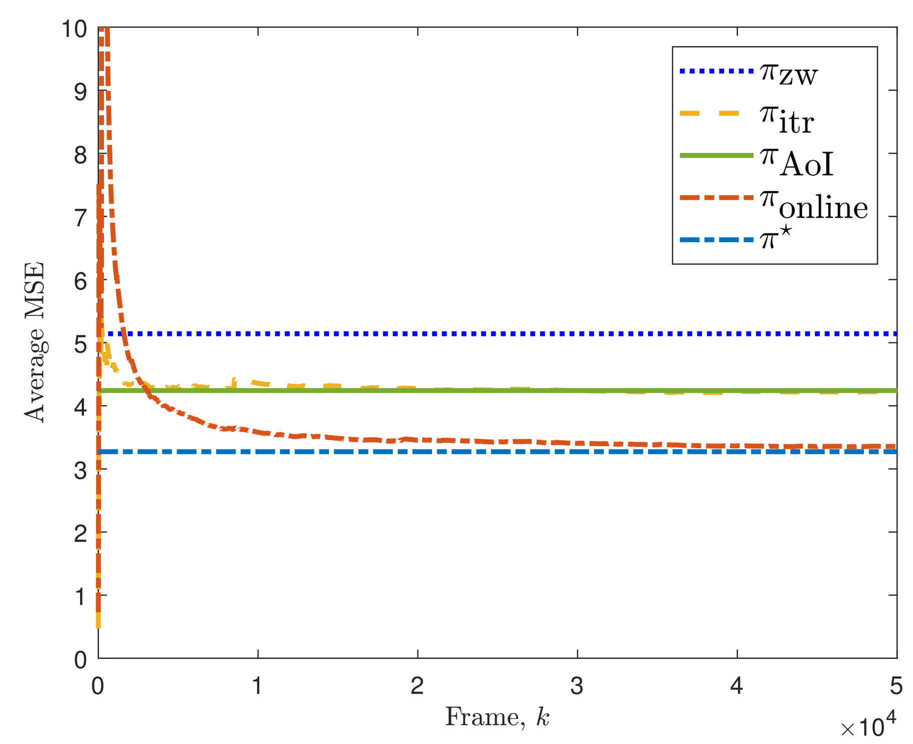
Theoretic Results

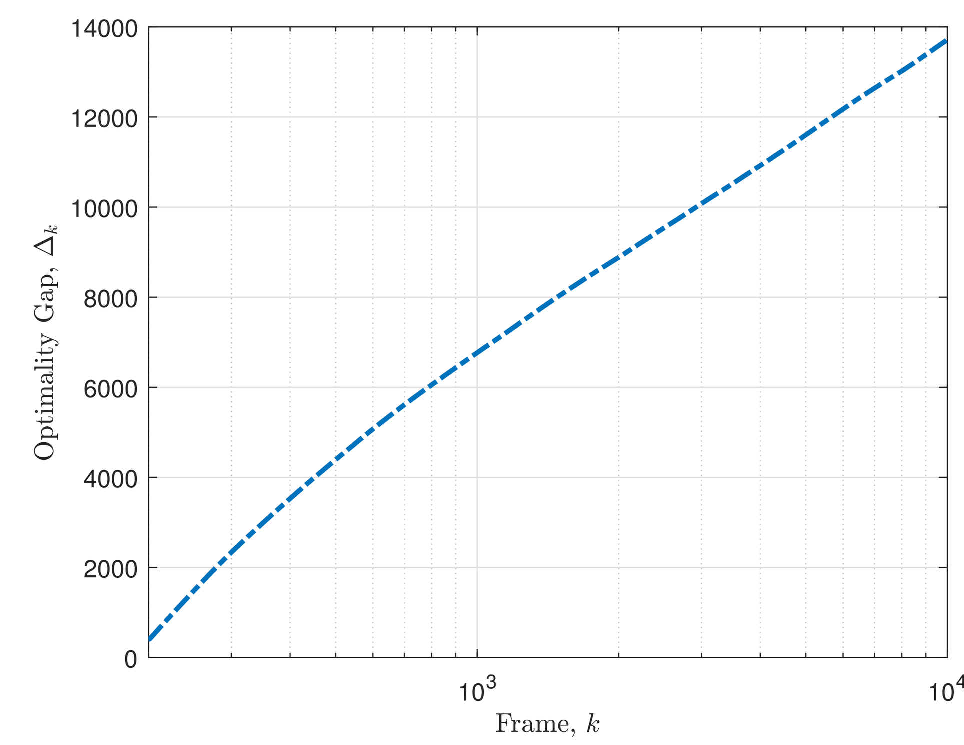
Theorem 1: (Convergence) If the delay D is forth-order bounded $\mathbb{E}[D^4] < \infty$, the time-average MSE π_{online} converges to $\mathsf{mse}_{\text{opt}}$ almost surely, and cumulative MSE regret gap up to frame $(K+1)$:

$$\Delta_K := \mathbb{E}\left[\int_{t=0}^{S_{K+1}}(X_t - \hat{X}_t)^2 \mathrm{d}t\right] - \overline{\mathcal{E}}_{\pi^\star}\mathbb{E}[S_{K+1}] = \mathcal{O}(\ln K).$$

Theorem 2: (Converse) For any causal policy π: $\inf_\pi \sup_{\mathbb{P}} \Delta_K = \Omega(\ln K)$.

Simulations





MSE evolution with frame $\mathbb{E}[\int_{t=0}^{S_{K+1}}(X_t - \hat{X}_t)^2\mathrm{d}t]/\mathbb{E}[S_{K+1}]$ (left); Regret Growth Rate $\Delta_k := \mathbb{E}\left[\int_0^{S_{K+1}}(\hat{X}_t - X_t)^2\mathrm{d}t\right] - \overline{E}_{\pi^\star}\mathbb{E}[S_{k+1}]$. (Right)