Scheduling to Minimize Age of Information in Multi-State Time-Varying Networks with Power Constraints

Haoyue Tang, Jintao Wang, Linqi Song and Jian Song

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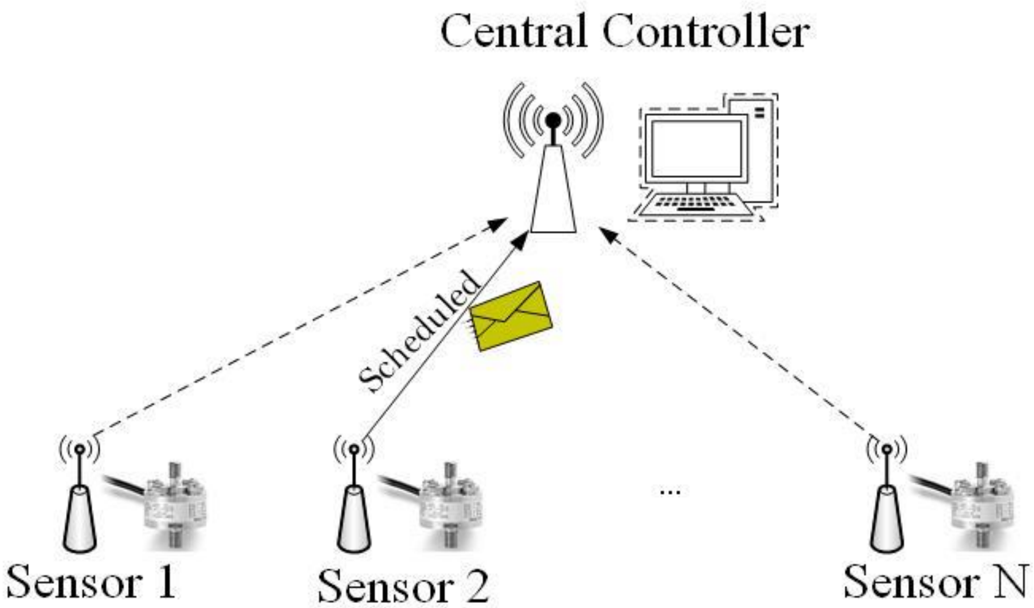
Outline

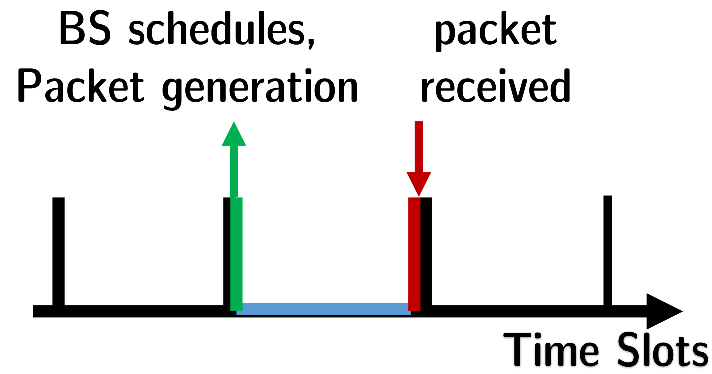
- Network Model
  - Age of Information
  - Problem Formulation
- **Problem Resolution**
  - **Decoupled-Single Sensor CMDP and LP resolution**
  - **Truncated Scheduling Policy and its asymptotic optimality**
- Numerical Simulations

Network Model - Overview



BS schedules to collect fresh information from power constrained sensors in a bandwidth limited time varying network

A discrete time scenario



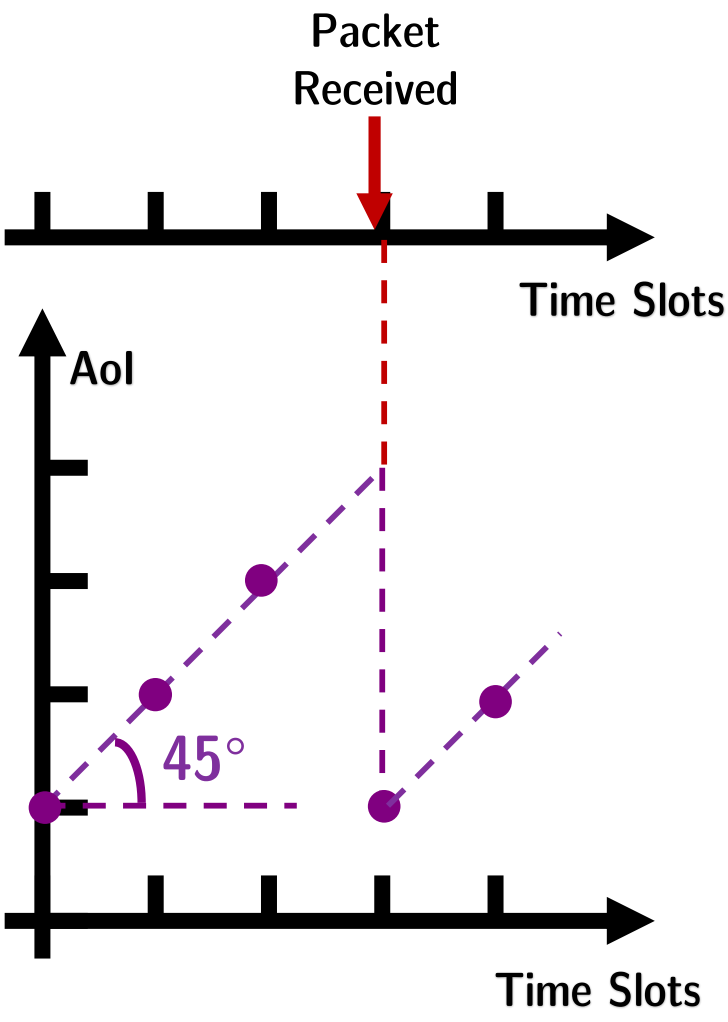
BS sends scheduling decision $u_n(t)$ at the beginning of slot t

- $u_n(t)=1$, BS schedules sensor n to send update, packet will be successfully received at the end of slot t
- $u_n(t)=0$, not scheduled

Network Model – Age of Information

- Def: AoI measures time elapsed since freshest information at the receiver was generated.

- Let $x_n(t)$ denote the AoI of sensor n at the beginning of slot t. Consider an error-free transmission:
  - If sensor n is not schedule, $u_n(t) = 0$, then $x_n(t+1) = x_n(t) + 1$
  - If sensor n is scheduled, $u_n(t) = 1$, then $x_n(t+1) = 1$



Literature

	Power Constraint		User Number		Channel Statistics	
Papers	Yes	No	Single	Multiple	Invariant	Variant
Sun18	✔		✔		✔	✔
Ceran18	✔		✔		✔	
Yang17	✔		✔		✔	
Yates17		✔		✔	✔(Perfect)	
He18		✔		✔	✔(Perfect)	
Hsu18		✔		✔	✔(Perfect)	
Kadota18		✔		✔	✔(i.i.d packetloss)	
Talak18		✔		✔		✔(Two-state)
Lu18		✔		✔		✔(Rate Varying)
This Work	✔			✔		✔(Multi-state)

Contributions: theoretic guarantees, asymptotic optimal strategy

Network Model – Constraints

- Time-Varying Channel States
  - ✓Channels are quantized into Q states, the link from user n to BS $q_n(t) = q$ w.p. $\eta_{n,q}$, successful transmission in state q takes $\omega(q)$ power.
- Bandwidth Limited Network:
  - ✓At each slot, no more than M sensors can be scheduled

$$\sum_{n=1}^{N} u_n(t) \leq M, \forall t.$$

- Power Constrained Sensors
  - ✓Scheduling decision must satisfy the average power constraint of each sensor

$$\frac{1}{T}\sum_{t=1}^{T} \omega\big(q_n(t)\big)u_n(t) \leq \mathcal{E}_n, \forall t.$$

Network Model – Problem Statement

- Goal: Design a non-anticipated scheduling strategy that minimize average AoI performance under both bandwidth and power constraints.

Problem 1: (B&P Constrained AoI)

$$\pi^* = \arg \min_{\pi \in \Pi_{\mathrm{NA}}} \lim_{T\to\infty} \frac{1}{NT} \mathbb{E}_\pi\left[\sum_{t=1}^{T} \sum_{n=1}^{N} x_n(t)\right],$$

$$s.t. \sum_{n=1}^{N} u_n(t) \le M, \text{(bandwidth)}$$

$$\lim_{T\to\infty} \frac{1}{T} \mathbb{E}_\pi\left[\omega\big(q_n(t)\big) u_n(t)\right] \le \mathcal{E}_n. \text{(power)}$$

- Challenges: Bandwidth constraint → NP-Hard Integer Programming

Problem Resolution

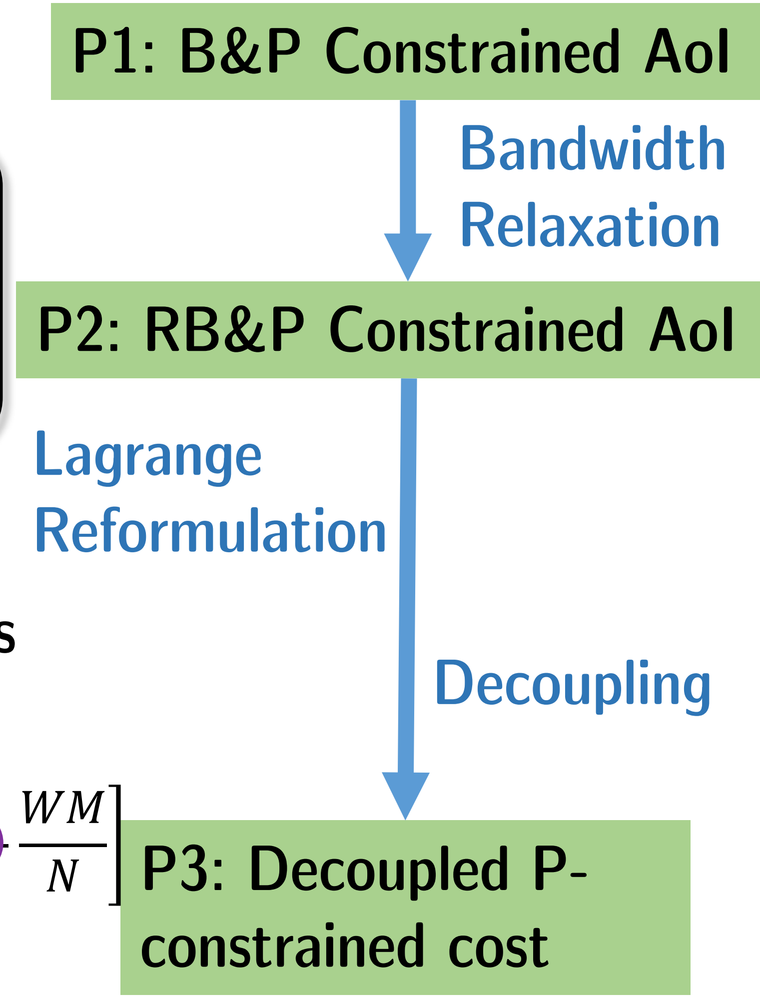
- <u>Step 1 Relaxation:</u> Time average number of sensors scheduled is smaller than M.

> Problem 2: (RB&P Constrained AoI) Minimizing AoI under power and relaxed bandwidth constraint:
> $$\frac{1}{T}\sum_{t=1}^{T}\sum_{n=1}^{N} u_n(t) \le M, \text{(relaxed bandwidth)}$$

- <u>Step 2 Lagrange Reformulation:</u> the optimum policy π_R to Problem 2 is a mixture of two policies π_W that minimizes the Lagrange function with no bandwidth constraint:

$$\pi_W = \arg\min_{\pi \in \Pi_{\text{NA}}} \mathbb{E}_\pi \left[\frac{1}{NT}\sum_{t=1}^{T}\sum_{n=1}^{N} (x_n(t) + W u_n(t)) - \frac{WM}{N} \right]$$

- <u>Step 3 Single Sensor Decoupling</u>



Problem Resolution – Decoupled Single Sensor CMDP

- Decoupling: Minimizing Lagrange function with no bandwidth can be solved separately for each sensor (hence omit subscript n)

Problem 3: (Decoupled P-Constrained Cost)

$$\pi_d = \arg\min_{\pi \in \Pi_{\mathrm{NA}}} \lim_{T\to\infty} \frac{1}{T} \mathbb{E}_\pi \left[\sum_{t=1}^{T} \big(x(t) + W u(t) \big) \right],$$

$$s.t. \lim_{T\to\infty} \frac{1}{T} \mathbb{E}_\pi \left[\omega\big(q(t)\big) u(t) \right] \le \mathcal{E}_n . \text{ (power)}$$

Theorem 1: The optimum strategy to Problem 3 is a stationary randomized policy, when the state $\big(x(t), q(t)\big)$ equals x, q, the optimum policy schedules with probability $\xi_{x,q}$.

Moreover, the optimum policy has a threshold structure, there exists a sequence τ_q:

$$\xi_{x,q} = 0, x < \tau_q; \xi_{x,q} = 1, x > \tau_q; \xi_{x,q} \in (0,1], x = \tau_q.$$

Problem Resolution – Linear Programming for Decoupled CMDP

- Given W, optimum policy π^W schedules the decoupled sensor when AoI=x, channel state=q w.p. $\xi_{x,q}^W$.
- Due to threshold structure, $\exists X, \xi_{x,q}^W = 1, \forall x \geq X$
- So what is $\xi_{x,q}^W$ for those $x < X$? → This can be solved through an LP.

- Finally, to satisfy relaxed bandwidth constraint, find two Lagrange multipliers W_l and W_u through dual method. The mixture of policy π^{W_l} and π^{W_u} schedules sensor n w.p. $\xi_{x,q}^{n,*}$ when his AoI equals x and channel state is q.

Problem Resolution – A Truncated Scheduling Algorithm

- Policy π_R^*: In each slot t, BS schedules sensor n w.p. $\xi_{x_n(t),q_n(t)}^{n,*}$

Policy π_R^* is the optimum policy of scheduling under relaxed bandwidth constraint, the average AoI performance $J(\pi_R^*)$ formulates the lower bound of scheduling with bandwidth constraint in every slot.

- Truncated scheduling policy $\hat{\pi}$. In each slot, sensor n is put into set $\Omega(t)$ w.p. $\xi_{x,q}^{n,*}$. Sensors in $\Omega(t)$ is eager to be scheduled.
  - If $|\Omega(t)| \leq M$, BS schedules all the sensors in $\Omega(t)$
  - Otherwise, BS schedules M sensors from $\Omega(t)$ randomly

Theorem 2: The proposed algorithm $\hat{\pi}$ is asymptotic optimal. Let M/N be a constraint and $\mathrm{AoI_{LB}}$ is the average AoI lower bound (π_R^*):

$$\frac{J(\hat{\pi}) - AoI_{LB}}{AoI_{LB}} = \mathcal{O}\left(\frac{1}{\sqrt{N}}\right), i.e., \lim_{N\to\infty} \frac{J(\hat{\pi}) - AoI_{LB}}{AoI_{LB}} = 0$$

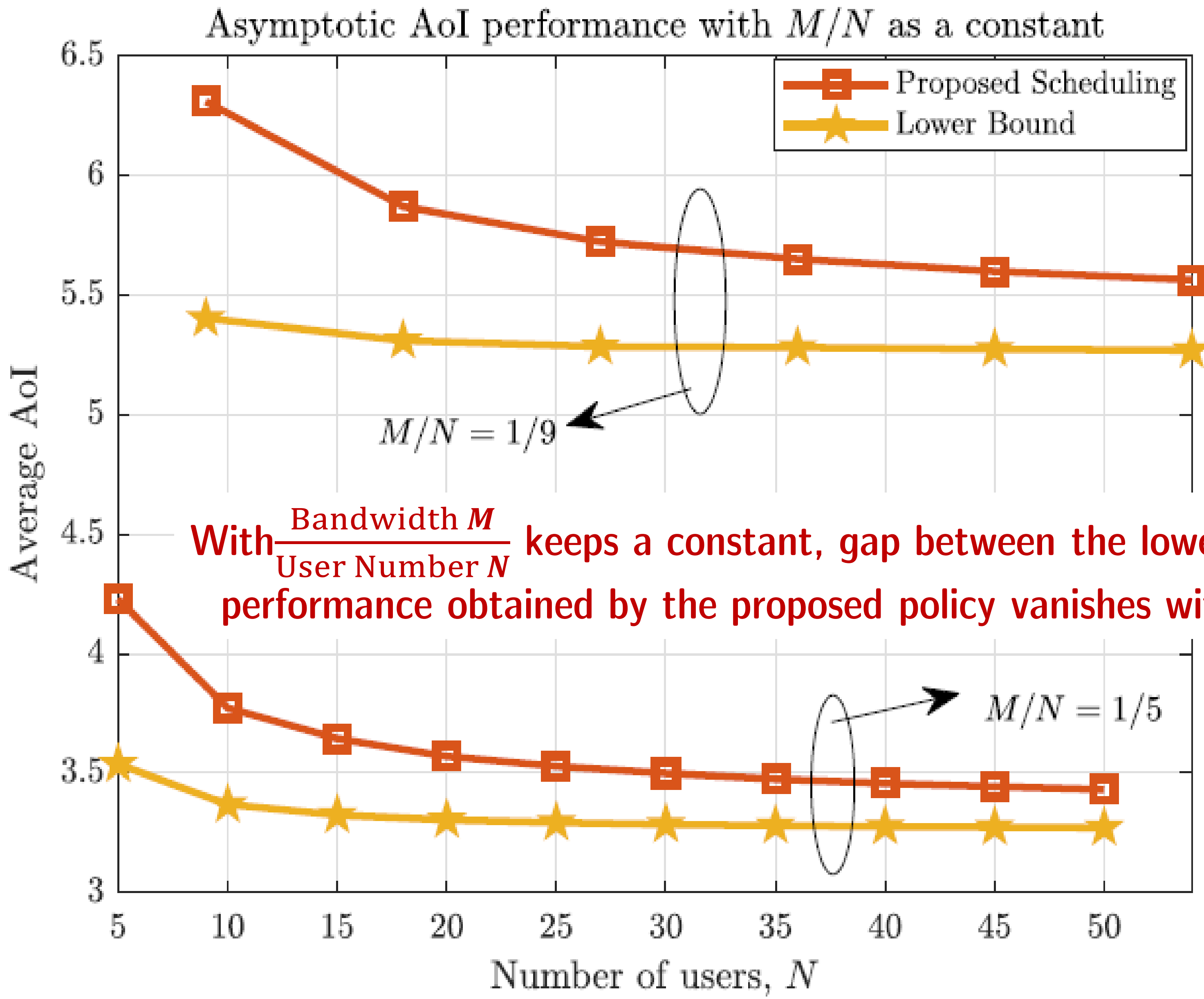
Simulations

- Metric: Average Age of Information over a consecutive of $T = 10^6$slots

- With no power constraint, Round Robin is optimum for identical sensor, requires a minimum average power of $\mathcal{E}_n^{RR} = \frac{M}{N}\sum_{q=1}^{W} \eta_{n,q}\omega(q)$

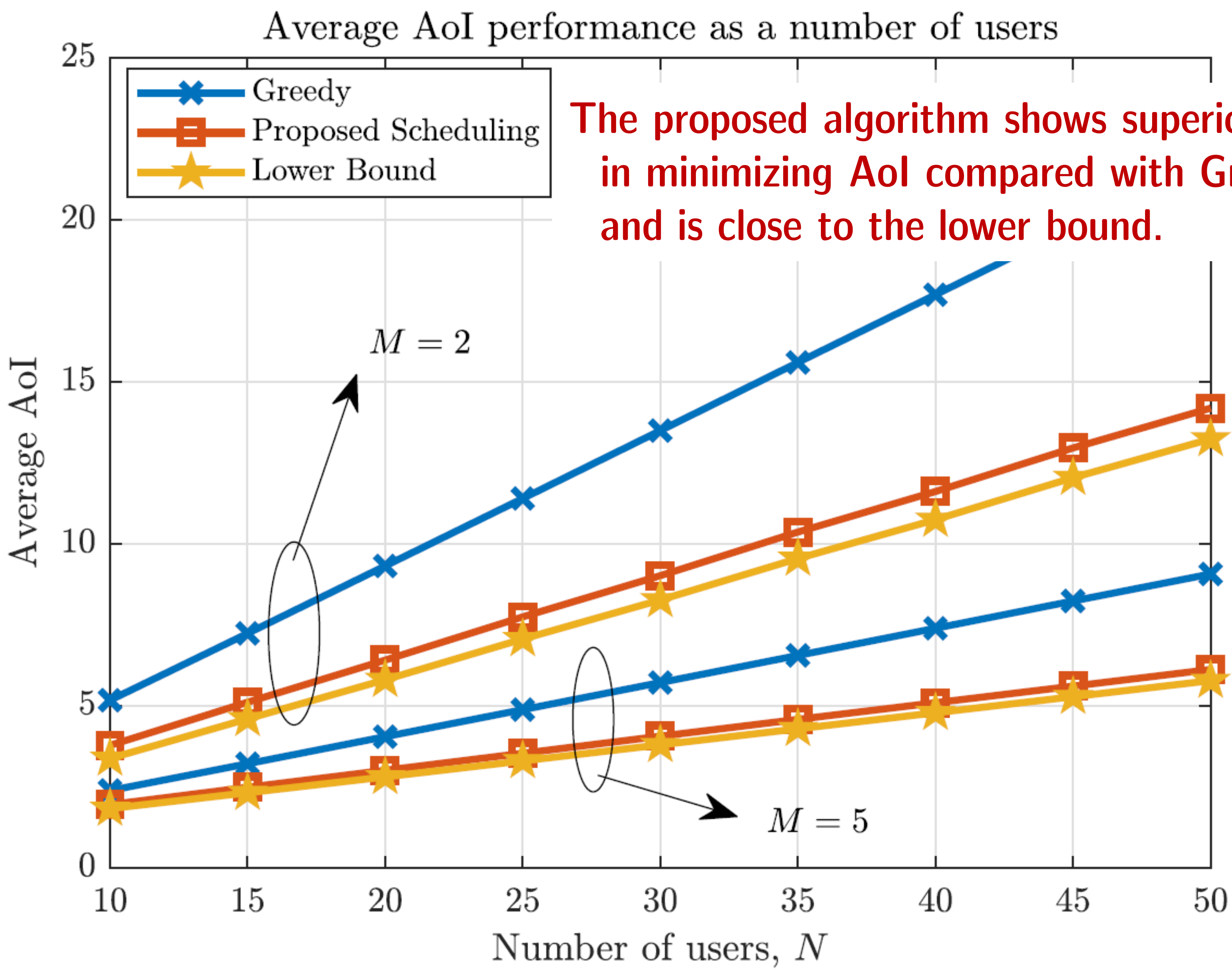
- Coefficients:
  - A $Q = 4$ quantized channel
  - Channel statistics $\eta = [0.135, 0.239, 0.232, 0.394]$ for every sensor n
  - Power consumption $\omega(q) = q$
  - Power constraint factor $\rho_n = \frac{\mathcal{E}_n}{\mathcal{E}_n^{RR}}$

Simulations 1: Asymptotic Analysis



With $\frac{\text{Bandwidth } \boldsymbol{M}}{\text{User Number } \boldsymbol{N}}$ keeps a constant, gap between the lower bound and the performance obtained by the proposed policy vanishes with $N \rightarrow \infty$.

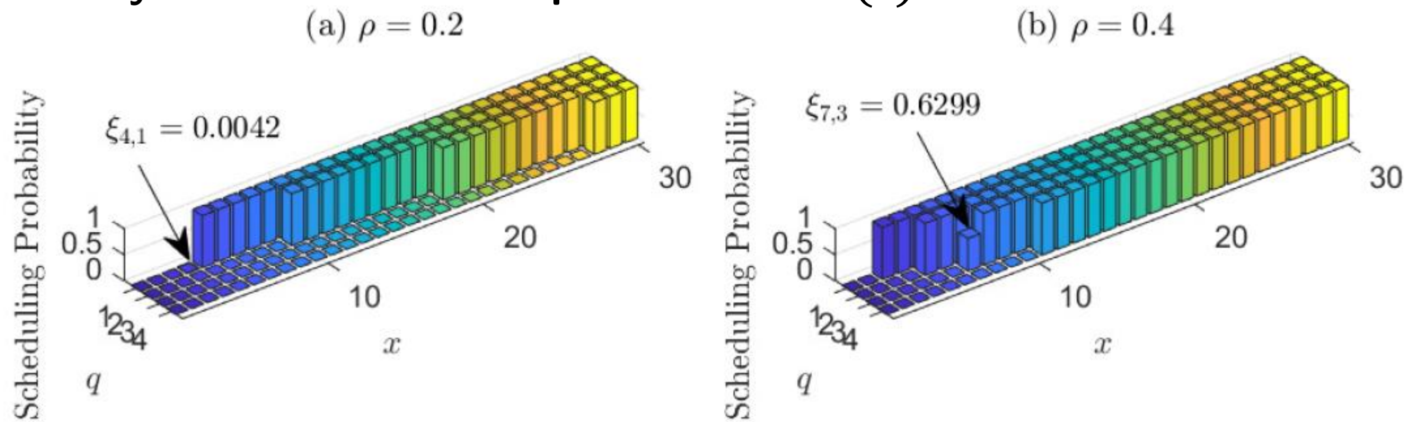
Simulations 2: Algorithm Comparisons



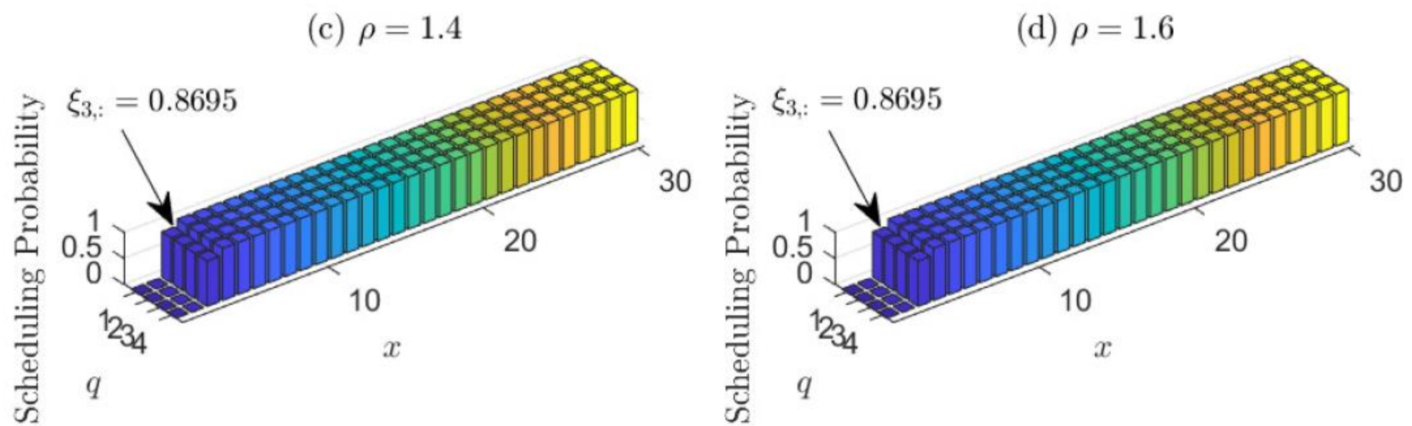
The proposed algorithm shows superior performance in minimizing AoI compared with Greedy algorithm and is close to the lower bound.

Simulations 3: Illustration of Scheduling Decisions

Scheduling decisions of different power constrained sensors in a network with $M = 2, N = 8$, channel statistics is the same as before, each sensor $\rho_n = 0.2n$. Illustrate the probability that the sensor is put into set $\Omega(t)$.



1. For power constrained sensors, schedule in better channel states or the AoI is large.



2. For sensors equipped with enough power, scheduling decision has to satisfy bandwidth constraint, scheduling thresholds in all channel states can be the same.

Final Remarks

- **In this talk:**
  - multi-user scheduling in time-varying networks under power and bandwidth constraint
  - LP approach to the decoupled single sensor
  - An asymptotic optimal truncated scheduling algorithm
  - Numerical Simulations: Proposed algorithm has superior performance

- **In this paper**
  - Detailed Proofs of optimum structure and asymptotic optimality

- **Recent On-going work:**
  - Channel evolution has Markov properties, packet-loss, etc.

Thank you! Q&A